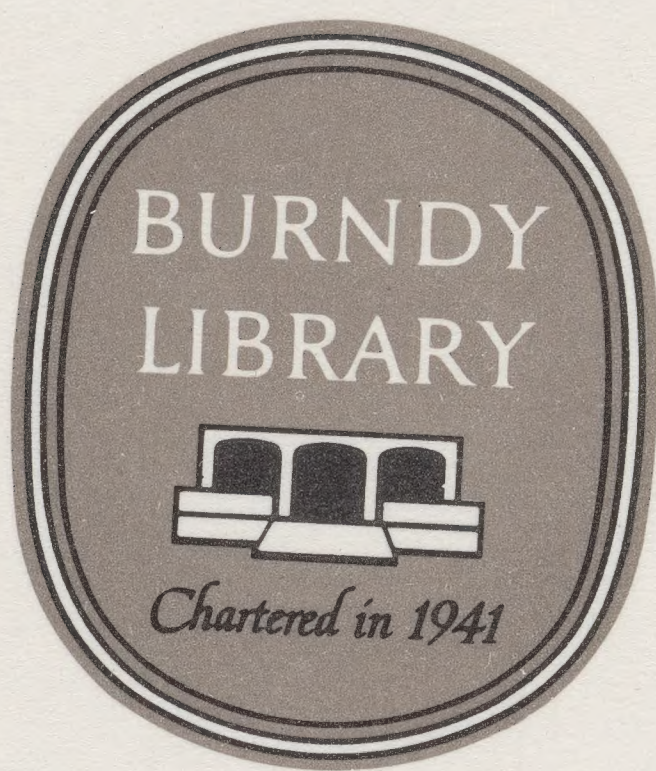


"Light"

Dr. Kimball

C. L. P.



Gift of
BERN DIBNER

Wave Motions

Light has a certain periodicity, this is shown by interference. It has a certain definite velocity and energy.

These are characteristic of wave motions. For motions of waves a medium is necessary, hence the hypothesis of the luminiferous ether. The nature of the disturbance propagated is not known; theory of electro magnetic disturbance. The wave disturbance is not guaranteed about the direction of propagation — polarized light. (Sound)

On the old theory of the ether as an elastic solid, the

DSI

waves were waves of rigidity.
In general there are two kinds
of waves, compressions and
rigidity. How to get rid of the
compression waves was impossible
on the old theory. Superior
velocity was not good, for it
fails in the case of double
refraction.

His ^{own} Thomson's idea of
the ether. Keeping to the old
elastic solid theory, but instead
of infinite elasticity, he has
no bulk elasticity. (From ex.)
This treatment seems to fit
mathematically the requirements

The electro-magnetic theory
does not encounter these
difficulties; it gives expressions
for reflection and double
refraction, and aside from
the artificiality of Sir Wm's theory

the coincidences are remarkable.

Light waves

Anal. to vibrating cord: —
medium composed of a
series of cords — the neglect
transversal elasticity.

Each particle performs
a simple harmonic vibration
by general a vibrating body
sending out light has not
the same period for all
particles — The motion is
white light will not in
general be elliptical but
highly irregular.

Fundamental objections
to wave theory removed by

Huygens and Fresnel

Given periodic vibration of any form - let y = displacement.

$y = \varphi(x, t)$
When the point at x' reaches x then y_1 becomes y , and if v is the velocity, and $t - t' =$ time of passage from x_1 to x , then

$$v(t - t') = x - x_1$$

$$y = \varphi(x, t) = \varphi(x', t') \quad (2)$$

Substituting $y = \varphi[(x + vt' - vt), t']$

but t is any time, and making

$$t' = 0$$

$$y = \varphi(x - vt)$$

Comparing (1) and (3) we may expand by Fourier's theorem in a series of sines & cosines. whence...

$$y = a + a_1 \sin \frac{2\pi}{\lambda} (vt - x) + \frac{a_2}{2} \sin \frac{2\pi}{\lambda} (vt - x) + \dots$$

where t - length of single complete vibration.

If we have the axis of x so placed that it cuts off equal areas above and below, then the first constant is zero.

When we make $t=0$ then all the harmonic motions pass thro' the origin; but we want to represent any motion.

If we put

$$\sin \frac{2\pi}{\lambda} (vt - x + b)$$

b = any arbitrary constant, we can cause the phase to be anything at the origin. Will show that the same result is obtained by adding the cosine terms.

i.e.

$$A_1 \sin a + B_1 \cos a = A \sin(a+b) \\ = A(\sin a \cos b + \cos a \sin b)$$

This will be satisfied if.

$$A_1 = A \cos b, \quad B_1 = A \sin b$$

$$A^2 = A_1^2 + B_1^2, \quad \tan b = \frac{B_1}{A_1}$$

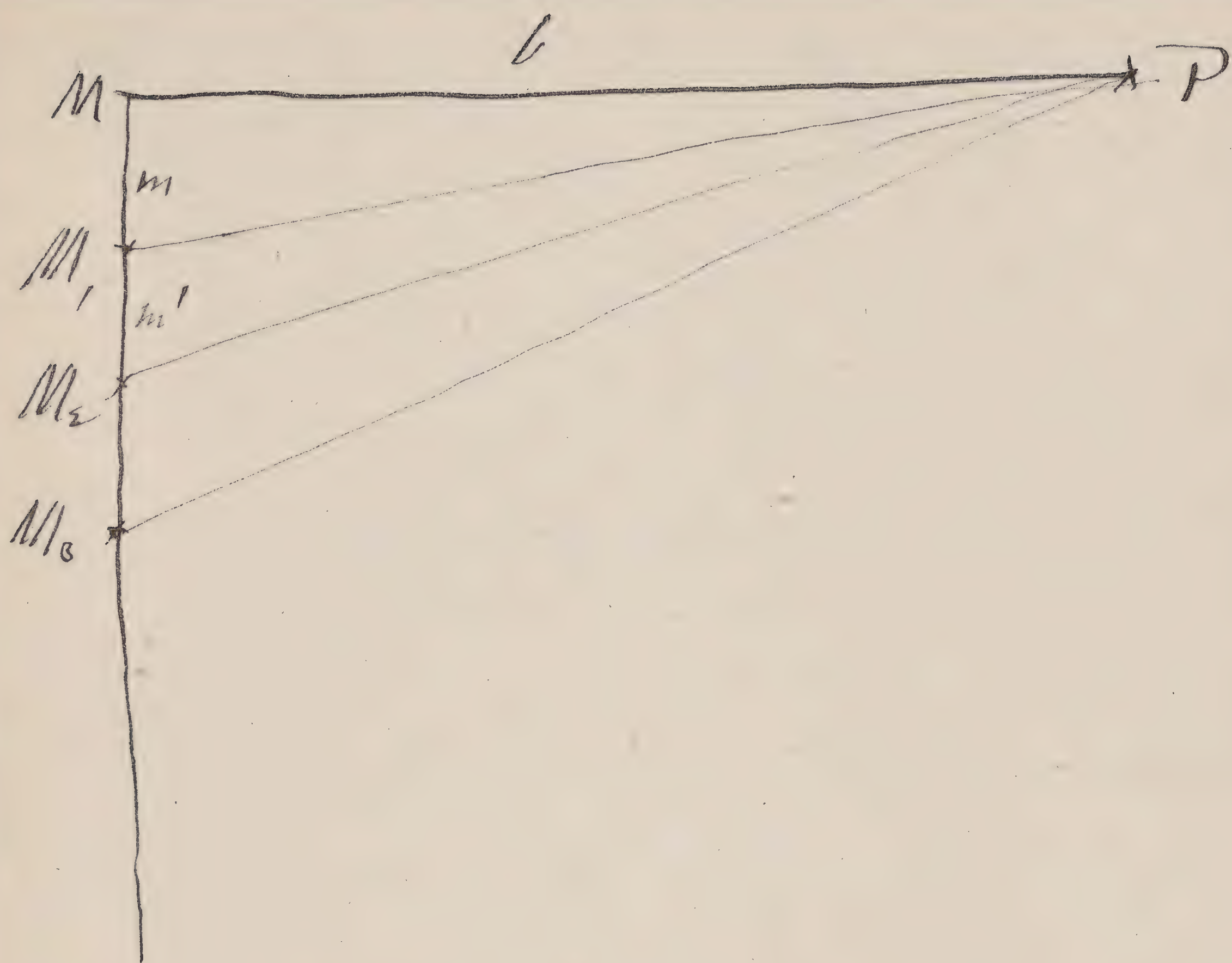
In our study of light
we shall deal with one of these
tones and call it light of
a definite wave length.

Fourier's method of
decomposing harmonic
disturbance is not the
only one. Each harmonic

act as a single mechanical unit.

If we have two substances, one half a wave length behind the other and of equal amplitude, it is obvious that we shall get complete interference.

We are now in a position to show that light travels in straight lines.



$$MM_1 = 1 = 1$$

$$M_1 M_2 = \sqrt{2} - \sqrt{1} = .414$$

$$M_2 M_3 = \sqrt{3} - \sqrt{2} = .318$$

$$\sqrt{1.5} - \sqrt{1.07} = .054$$

$$\sqrt{\quad} = .049$$

The wave from M and that from M₁ will reach the point P in exactly opposite phase. The total disturbance

at P may be taken as the
sum of those due to the
elements. taken with appropriate
signs.

$$M - m_1 + m_2 - m_3 + m_4 \text{ etc.}$$

For light in the brightest
part of the spectrum, and
 $t = 1 \text{ in.}$ the distance $M - \frac{M}{100}$
 $= \frac{1}{100} \text{ in.}$ A screen $\frac{1}{100}$
of an inch wide at M would
then cut off all light at P;
this account for shadows,
for what we mean by saying
light travels in str. lines.

Huyghens principle

At any instant all parts of a spherical wave give rise to spherical waves wh. as they increase in size, have a common ^{spherical} envelope wh. is a sph. surface, having its center at the original source of light and within env. all the details exist:

All parts of the wave within the sphere, completely interfere and no effect results. The only effective vibrations are on the wave surface —

That part of the wave front is effective at any

point, for which the light comes
in a minimum or max.
time. In an isotropic
medium this is the
minimum distance.



In many media the
wave travels faster in
some directions than in
others. In these cases
we must fall back on
the general principle of
least time.

Continue (ms) and laws of
reflection and refraction -
see Gla -

Then we have

$$\sin \theta' = \frac{1}{\mu}$$

then is total reflection.

The wave length depends
upon the period and the
medium. The period does
not change in passing
from one medium to another.

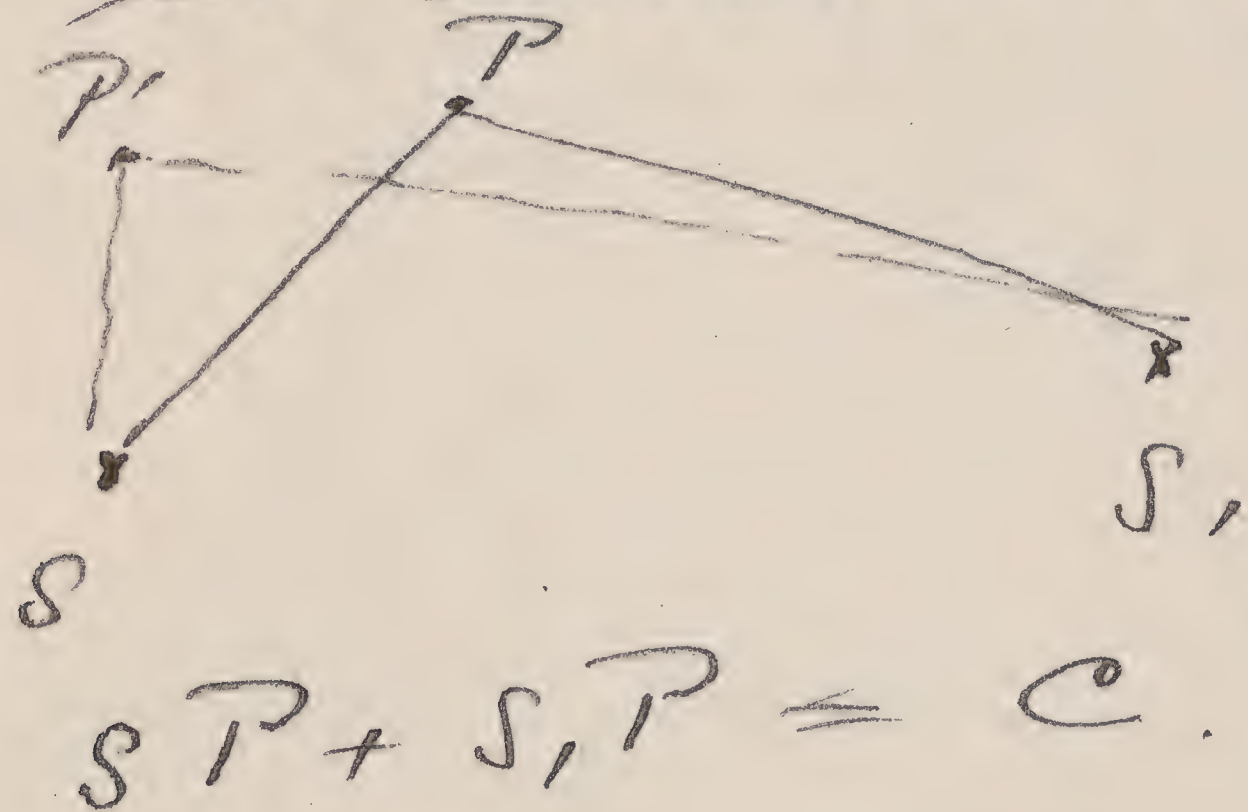
light passing thro' is
differently refracted. We have
anabys. of prism according
to wave length -

The index of refraction
is the ratio of the two velocities.
Air has an index of refraction
compared with vacuum.

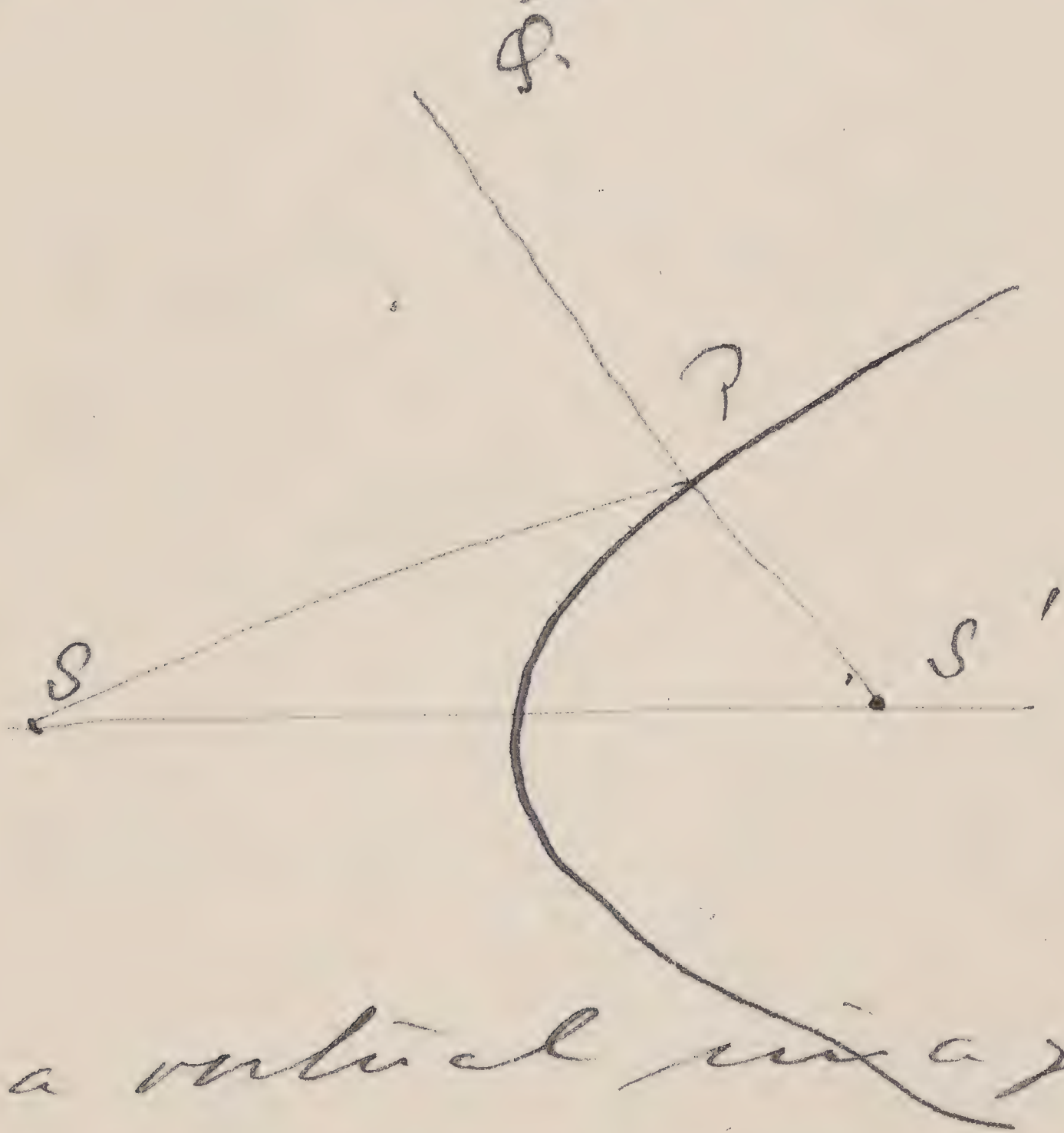
In the case of light
in the ether or vacuum, the
velocity is the same for
the different wave lengths.
Variable stars - Algol -

Light is to come from
S and be concentrated
at S'; what is the form
of surface that will do this.
The time for all rays is

the same - In the case of reflection, the light always is in same medium, its velocity is constant, $\therefore$ the path is constant.



the equation of an ellipsoid.



In a virtual image.

$$SP + PQ = c \quad S'P + PQ = c'$$

S' must be the center of wave

$S'Q$ is the radius -

$$SP - S'P = C$$

are hyperboloid.

Case of refraction



The time from S to S' is constant.

$$\frac{SP}{v} + \frac{S'P}{v'} = C$$

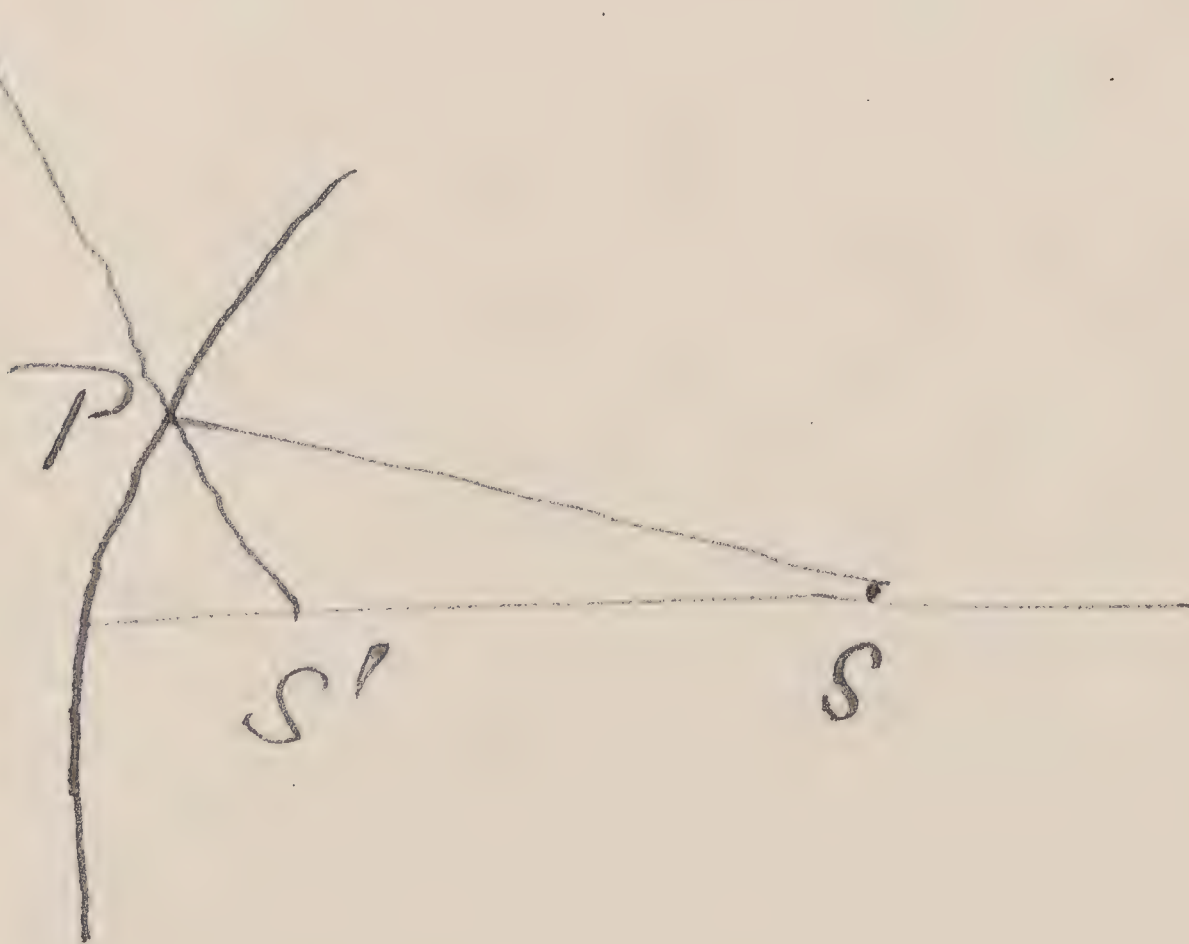
$$\therefore SP + \mu S'P = C$$

virtual focus

$$\frac{SP}{v} - \frac{S'P}{v'} = C$$

$\therefore$

$$SP - \mu S'P = C$$

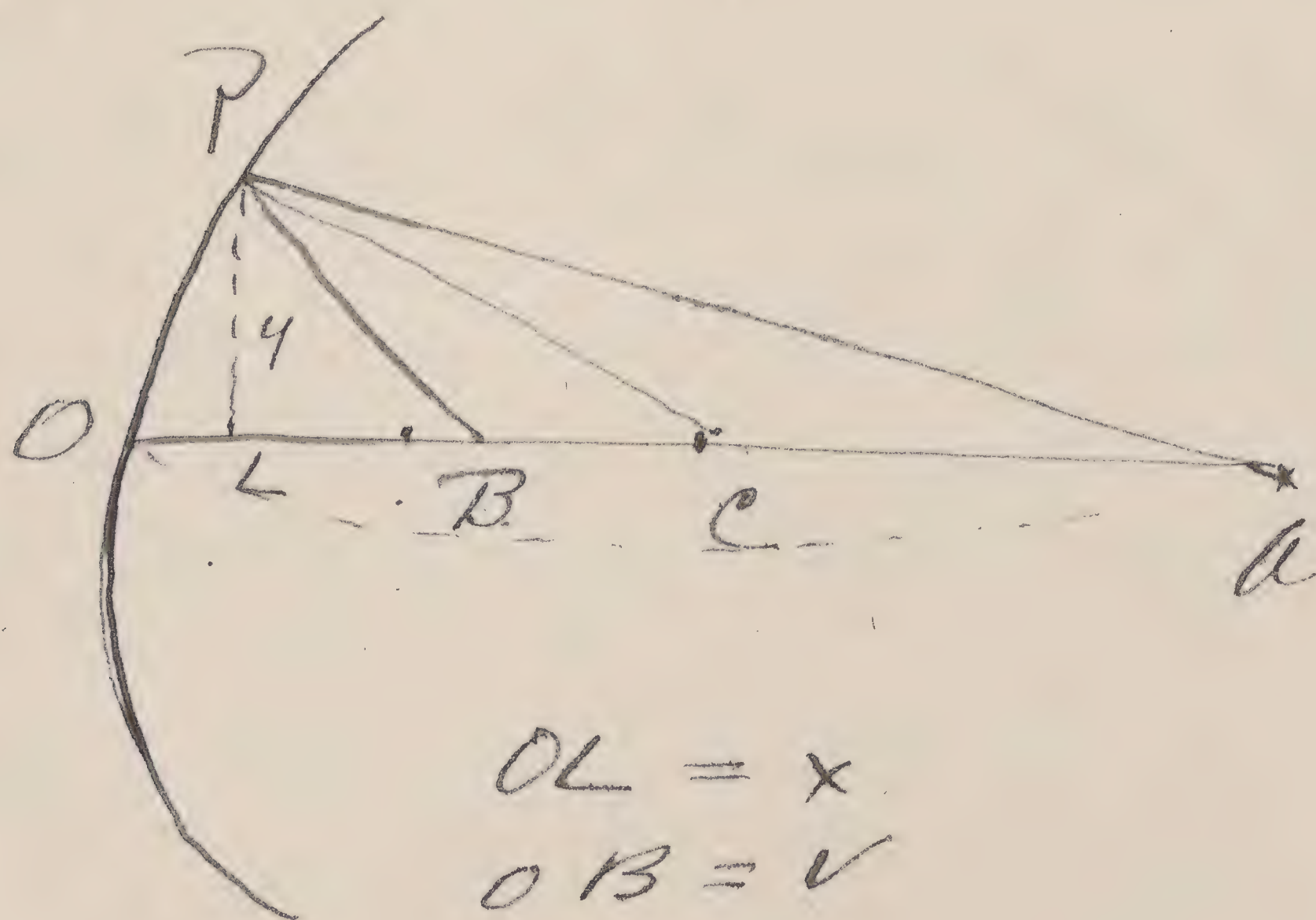


These equations are Cartesian
ovals -

In cases of refraction in
optical instruments, we use
spherical surfaces. Spherical
Aberration

These surfaces we have
found are aplanatic; a
spherical wave is after
deviation a spherical wave.

Reflection at spherical
mirr.



$$\begin{aligned} OL &= x \\ OB &= r \\ OA &= u \end{aligned}$$

neglecting x^2

$$x = \frac{y^2}{2r}$$

$$PA + PB = \text{const} = u + v$$

$$PA = \left[y^2 + \left(u - \frac{y^2}{2r} \right)^2 \right]^{1/2}$$

neglecting the first power of y

$$= \left[u^2 + uy^2 \left(\frac{1}{u} - \frac{1}{r} \right) \right]^{1/2}$$

$$= u + \frac{y^2}{2} \left(\frac{1}{u} - \frac{1}{r} \right)$$

$$u + v = u + \frac{y^2}{2} \left(\frac{1}{u} - \frac{1}{r} \right) + v + \frac{y^2}{2} \left(\frac{1}{v} - \frac{1}{r} \right)$$

hence

$$\frac{1}{u} + \frac{1}{v} = \frac{2}{r}$$

the equation of condition
that the light from A shall
be concentrated at B —
There is nothing that restricts
us to one value of u —

For the case of refraction at a single surface we have.

$$u - \mu v = C = PA - \mu PB \\ = u + \frac{y^2}{2} \left(\frac{1}{u} - \frac{1}{r} \right) - \mu \left[v + \frac{y^2}{2} \left(\frac{1}{v} - \frac{1}{r} \right) \right]$$

$$\frac{1}{u} - \frac{1}{r} = \mu \left(\frac{1}{v} - \frac{1}{r} \right)$$

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r}$$

Condition for refraction at a single surface.

Refraction thro' a lens.

replace v by K , and let v , distance of final focus from second surface - thickness of lens = t .

Then μ what we have already found

$$\frac{\mu}{K} - \frac{1}{u} = \frac{\mu-1}{r_1}$$

for second surface

$$\frac{\frac{1}{\mu}}{v} - \frac{1}{K+t} = \frac{\frac{1}{\mu}-1}{r_2}$$

between these two eqns
eliminate K ,

$$K = \frac{\mu u r_1}{u(\mu-1) + r_1}$$

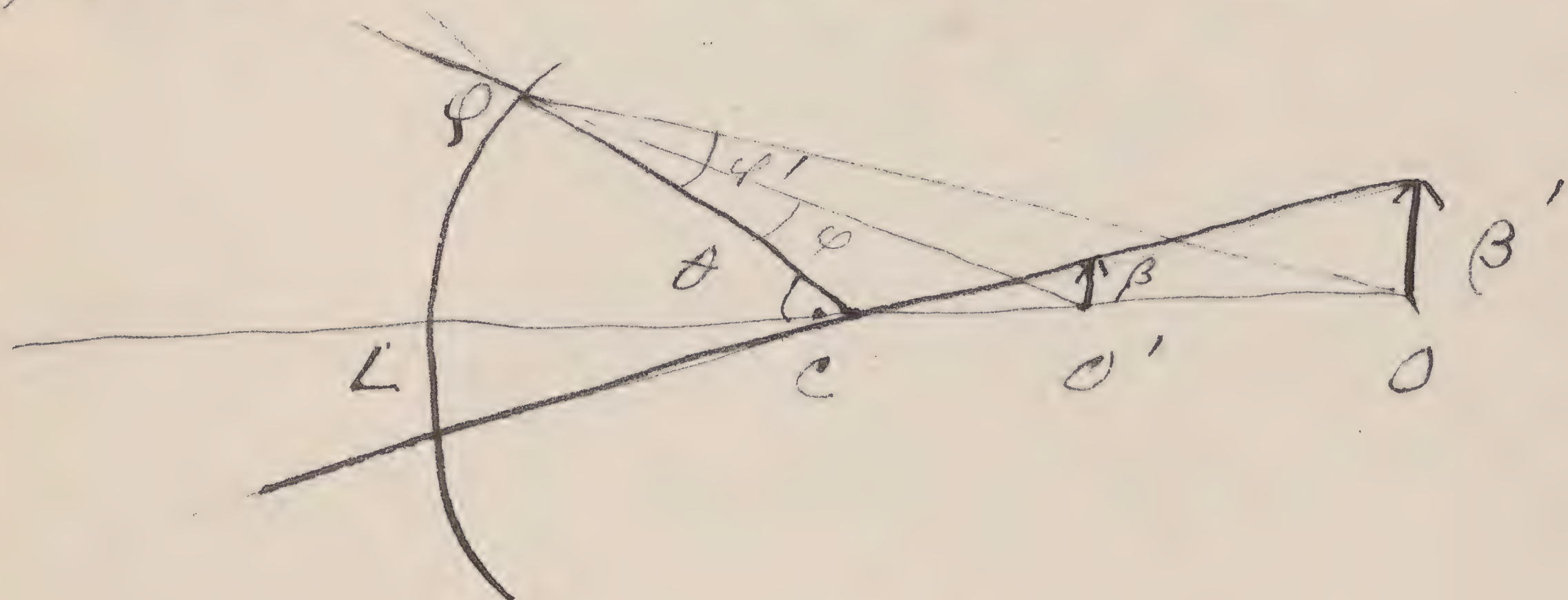
$$K+t = \frac{\mu v r_2}{v(\mu-1) + r_2}$$

$$t = \frac{\mu v r_2}{v(\mu-1) + r_2} - \frac{\mu u r_1}{u(\mu-1) + r_1}$$

If t is zero

$$\frac{1}{v} - \frac{1}{u} = (\mu-1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$$

Relation between size
of object and image, in
refraction at one surface



$$\mu \sin \phi = \mu' \sin \phi'$$

$$\frac{\sin \theta}{\sin \phi} = \frac{O'Q}{O'C}$$

$$\frac{\sin \theta}{\sin \phi'} = \frac{OQ}{OC}$$

$$\mu \frac{O'C}{O'Q} = \mu' \frac{OC}{OQ}$$

Now, take $OQ, O'Q$ as $OL, O'L$
 $= x', x$

$$\frac{\beta'}{\beta} = \frac{OC}{O'C}$$

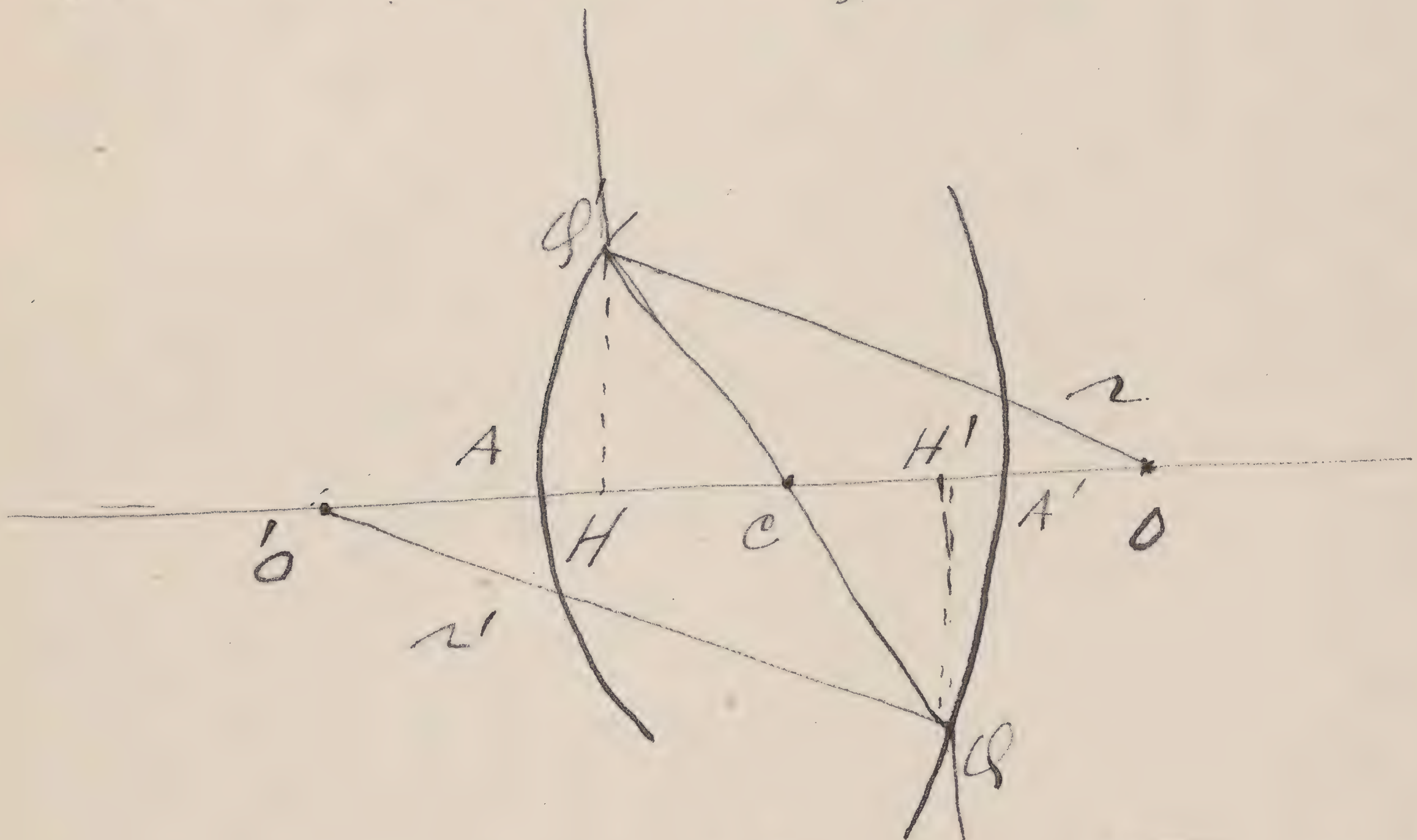
$$\mu \frac{\beta}{x} = \mu' \frac{\beta'}{x'}$$

$$\mu \beta \tan \alpha = \mu' \beta' \tan \alpha'$$

Position of principal planes,
nodal points for a single
lens -

$$\frac{\mu}{v} - \frac{1}{u} = \frac{\mu - 1}{r} = \frac{1}{f}$$

Take a thick lens of
different radii of curvature



C is called the center of the
lens - H and H' are
the nodal points. rays
in one medium proceeding
towards a nodal point, proceed
in the other as though they
came from the other node &

in a // direction to front rays.

$$O'C : OC = r' : r$$

The thickness of the lens.

$$t = ce\mu$$

$$OO' = r + r' - \mu c$$

$$O'C = \frac{r'}{r+r'} (r + r' - \mu c)$$

$$A'C = r' - O'C = \frac{\mu c r'}{r+r'}$$

$$AC = \frac{\mu c r}{r+r'}$$

$$A'C = \frac{\mu c f'}{f+f'}$$

$$AC = \frac{\mu c f}{f+f'}$$

$$h = AH; \quad h' = H'A'$$

$$\frac{\mu}{AC} - \frac{1}{h} = \frac{1}{f}$$

substitute.

$$\frac{f+f'}{cf} - \frac{1}{h} = \frac{1}{f}$$

$$h = \frac{cf}{f+f'-c}$$

and similarly

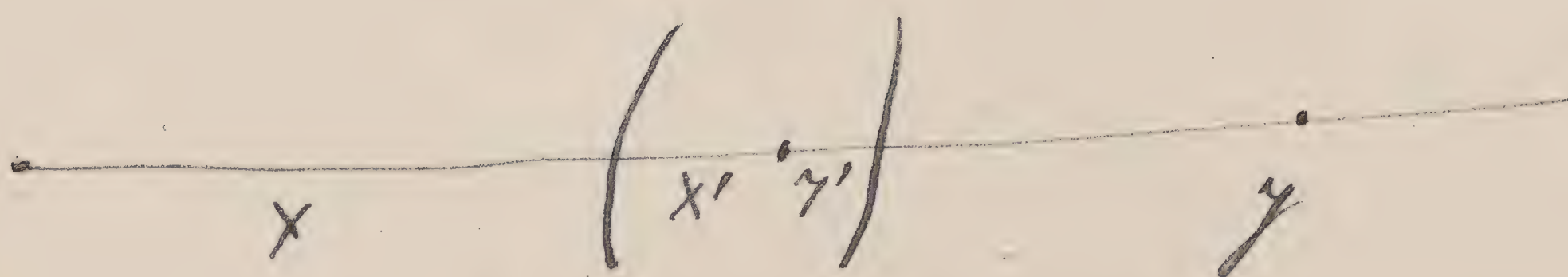
$$h' = \frac{cf'}{f+f'-c}$$

Principal planes are conjugate planes, in which the image and object have the same size and position. The points where these planes intersect the axis are principal points.

$$\frac{1}{x} + \frac{\mu}{x'} = \frac{1}{f} = \frac{\mu-1}{r}$$

$$\frac{1}{y} + \frac{\mu}{y'} = \frac{1}{f'} = \frac{\mu-1}{r'}$$

$$x' + y' = \mu c$$



The equation expressing the relation between the size of the image and object.

$$\frac{\beta}{x} + \frac{n\beta'}{x'} = 0$$

$$\frac{\beta''}{y} + \frac{n\beta'}{y'} = 0$$

where β'' is the size of the final image.

$$\frac{\beta}{\beta''} = \frac{xy'}{yx'}$$

and if the images are =
this ratio is = unity.

At the nodal points we
have $\frac{x'}{y} = \frac{n}{n'}$

$$c = \frac{1}{\frac{1}{f} - \frac{1}{x}} = \frac{1}{\frac{1}{f'} - \frac{1}{x'}}$$

$$f + x' - c = \frac{f}{x} (x' - c) - \frac{x'}{f} (f - c) = \frac{cf}{xx'} \quad (c)$$

for the principal focus:

$$g = f + f' - c$$

$$f + x' - c$$

$$g' = \frac{f' - c}{f + f' - c}$$

But we have found for the nodal points

$$h = \frac{cf}{f + f' - c}$$

$$h' = \frac{cf'}{f + f' - c}$$

Comparing these with the above

Convex



Concave





$$z + i = g' + i' = \rho = \frac{1^2 s'}{s + s' - c}$$

$$\text{dividing by } c \rightarrow z' + i' = c$$

$$xy - yg - xg' = c\phi$$

adding g, g' to both members
and factoring -

$$(x - g)(g - g') = \frac{H}{P} \left[\frac{(s' - c)(s - c)}{s + s' - c} + c \right]$$

$$= g^2$$

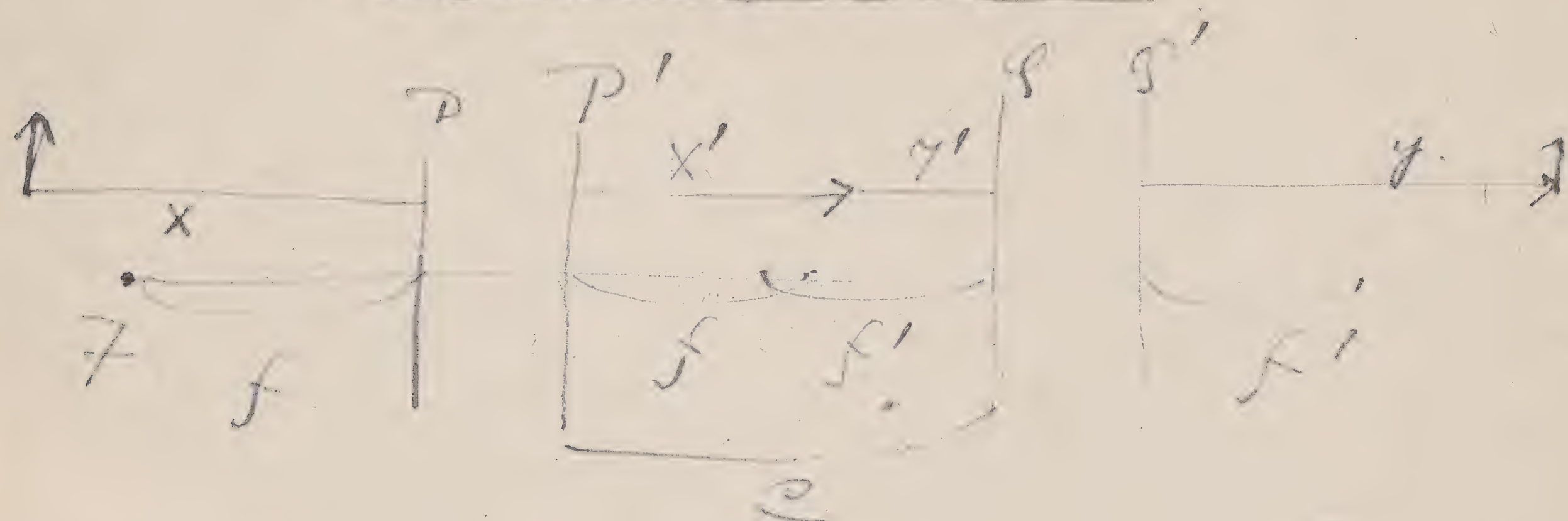
the above expression
neglecting the thickness of the
lens is

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{f}$$

g = distance from the principal
focus to the corresponding
nodal point -

$$\frac{1}{p + b} + \frac{1}{p + g} = \frac{2p + p + g}{p^2 + pg + gp + pg} = \frac{1}{p}$$

System of lenses



P, P', Q, Q' are the principal planes of the two lenses.

$$\frac{1}{x} - \frac{1}{x'} = \frac{1}{f}$$

$$\frac{1}{y'} + \frac{1}{y} = \frac{1}{f'}$$

$$x' + y' = c$$

eliminate x' and y' from these

$$c = \frac{1}{\frac{1}{f} - \frac{1}{x}} + \frac{1}{\frac{1}{f'} - \frac{1}{y}}$$

exactly the same form as before -

$$f = \frac{f f' - c}{f + f' - c}$$

$$f' = \frac{f f' - c}{f + f' - c}$$

g is measured from the same point as x , g' as y .

The equation for magnification

$$\frac{x\beta}{x} + \frac{x'\beta'}{x'} = 0$$

$$\frac{\beta}{x} = -\frac{\beta'}{x'}$$

$$\frac{\beta''}{\gamma} = -\frac{\beta'}{\gamma'}$$

$$\frac{\beta}{\beta''} = \frac{\gamma'\gamma}{x'\gamma}$$

$$h = \frac{c\gamma}{f + \gamma'c}$$

$$\gamma'c = \frac{c\gamma'}{f + \gamma'c}$$

given the principal planes as measured from P and c'

$$\varphi = g - h = g' - h' = \frac{f\gamma'}{f + \gamma'c}$$

the plane and nodal
 joints do not always coincide.
 When the third medium
 is different from the first,
 also in the eye, they do
 not coincide.



Prob. - find positions
 of principal planes, nodal
 joints etc for a Huygens -
 eye piece.



front lens of 1, is 3 in (2)
 and distance apart = $\frac{2}{3}$ focal
 length of longer

Brightness of image.

All the light wh. passes thro' the object glass passes thro' the ocular ring.

B = light. of object.

$B' =$ " " image.

S = surface of obj.

$S' =$ " " image.

d = distance of obj.

p = radius of pupil of eye.

r = " " ocular ring.

L = light received; direct vision.

$$L = \frac{BS}{d^2} \pi p^2$$

and light received.

$$L' = \frac{BS}{d^2} \pi R^2$$

where R is radius of lens.
glass.

Light entering eye;

L'' = light entering eye

If $p > r$, eye receives all
the light. — But if $p < r$,

$$\text{then } \frac{L''}{L'} = \frac{p^2}{r^2}$$

and magnifying power

$$\text{is } \frac{R}{r}$$

$$\frac{S}{S'} = \frac{r^2}{R^2}$$

$$L'' = \frac{p^2}{r^2} L' = \frac{p^2}{r^2} \cdot \frac{BS}{d^2} \pi R^2$$

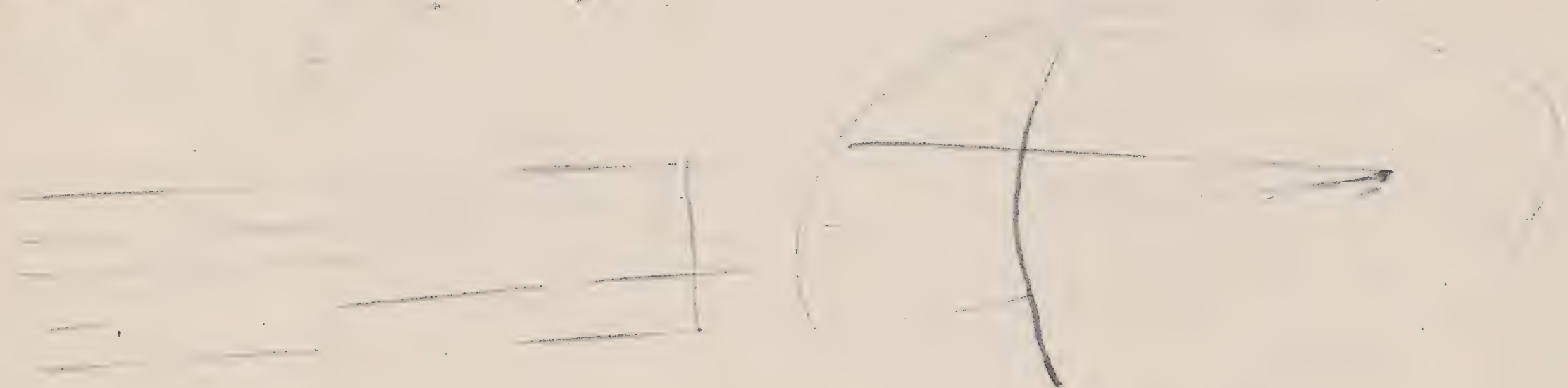
$$= \frac{BS'}{d^2} \pi p^2$$

the light received by the eye
as seen thro' the telescope.

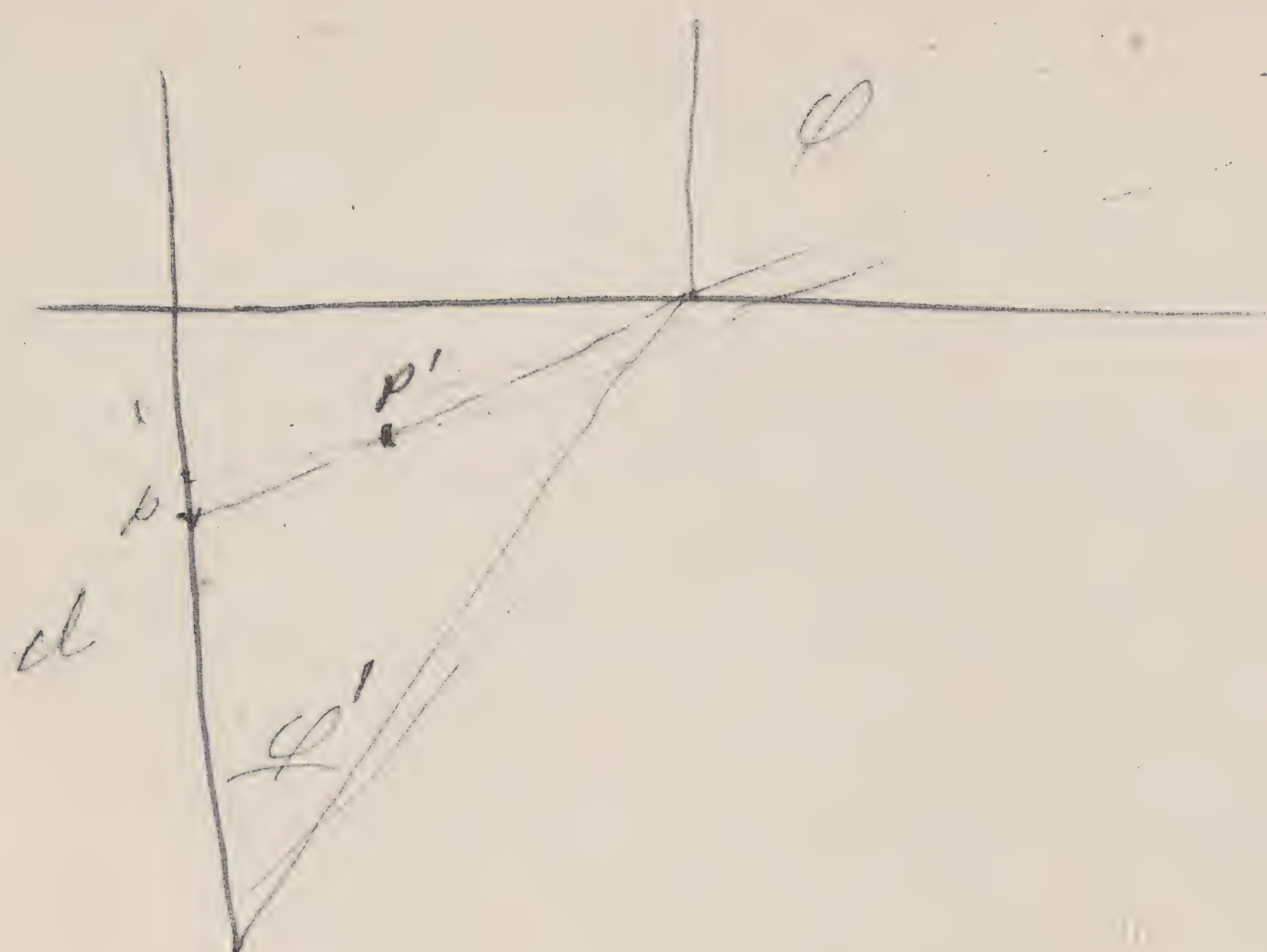
$B = B'$, the image can
never be brighter than the
object.

Sphaerocarpus

Can make a perfect
aplusid - same for
11' beam of light -



See surface - it can
slip out, the other a
slip out surface having
to enter the focus of
the slip out



$$\frac{\sin \phi}{\sin \phi'} = \mu$$

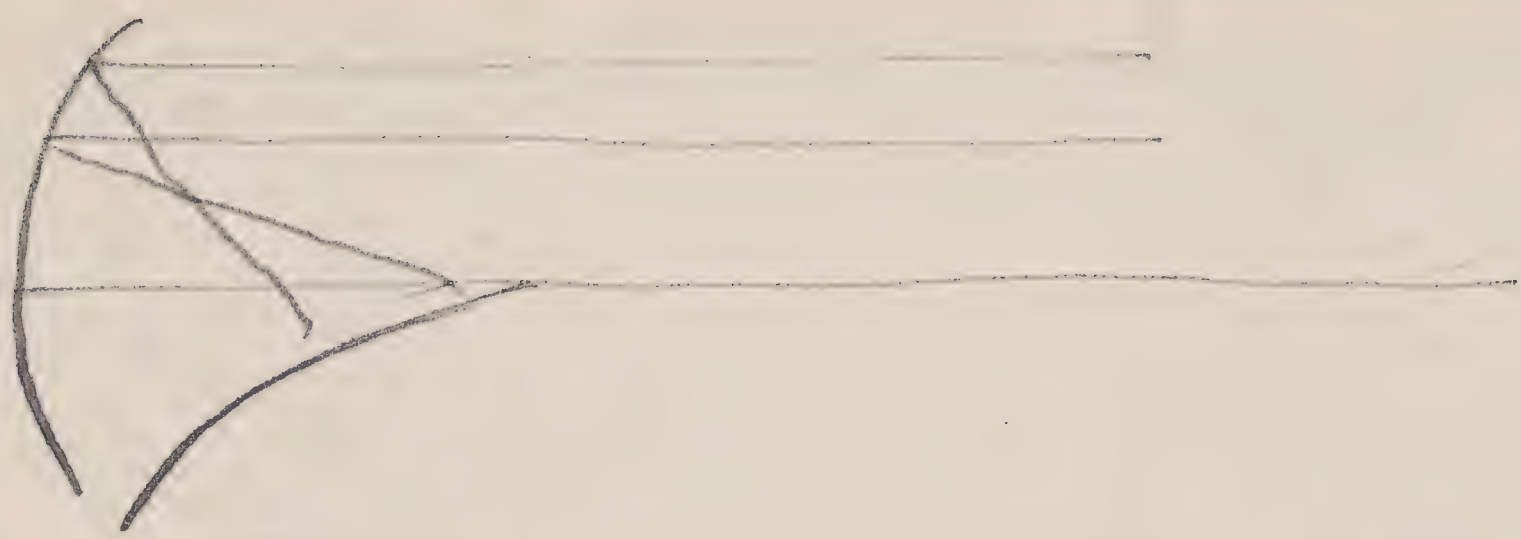
$$d \sin \phi' = x \sin \phi$$

$$d \frac{\sin \phi'}{\sqrt{1 - \sin^2 \phi'}} = x \frac{\mu \sin \phi'}{\sqrt{1 - \mu^2 \sin^2 \phi'}}$$

$$x = \frac{d}{\mu} \sqrt{\frac{1 - \mu^2 \sin^2 \phi'}{1 - \sin^2 \phi'}}$$

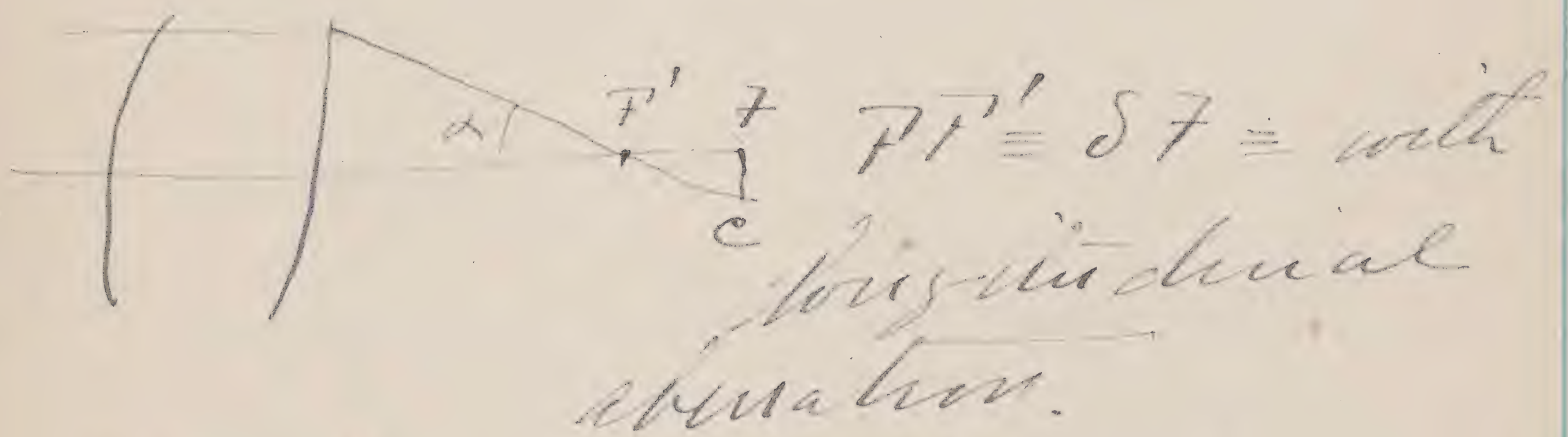
focal line L on axis
and circle at P' at
right angles to PL .

We may call the circle between
the two focal lines the circle
of minimum disturbance



The curve of spherical aberration is the caustic.

So in lenses when the light passes at a distance from the axis we have this spherical aberration which cannot be entirely gotten rid of.



$$\delta C \equiv \text{transverse aberration} \\ = \delta f \tan \alpha.$$

A parallel beam of light falling upon a lens gives the following relation

between focal length and
radius of curvature in
order that aberration may be
a minimum.

$$r = \frac{2(\mu+2)(\mu-1)}{\mu(2\mu+1)} f$$

$$-s = \frac{2(\mu+2)(\mu-1)}{4+\mu-2\mu^2} f$$

The spherical aberration
can never be made zero
for a single lens.

If we take $\mu = \frac{3}{2}$, as
it is nearly in glass, then

$$r = \frac{7}{12} f, \quad -s = \frac{7}{2} f$$

$$-\frac{s}{r} = +6; \quad \partial f = -\frac{15}{14} \frac{r^2}{f}$$

If we take the case
of a plano-convex lens and

substitute, making s the plane surface

$$-s = \infty$$

$$r = \frac{f}{2}$$

Then

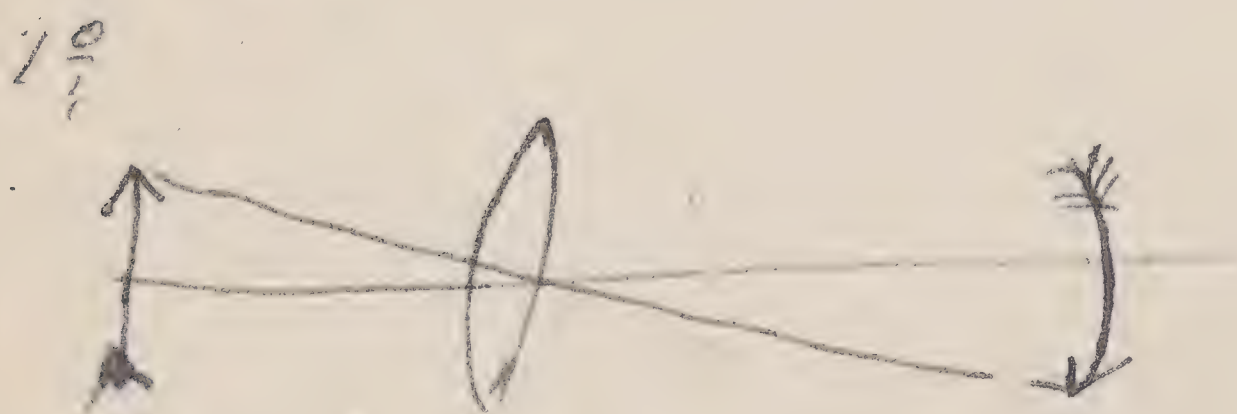
$$\delta f = -\frac{7}{6} \frac{r^2}{f}$$

If we make $r = \infty$ then

$$s = \frac{1}{2}f \text{ and } \delta f = -\frac{9}{2} \frac{r^2}{f}$$

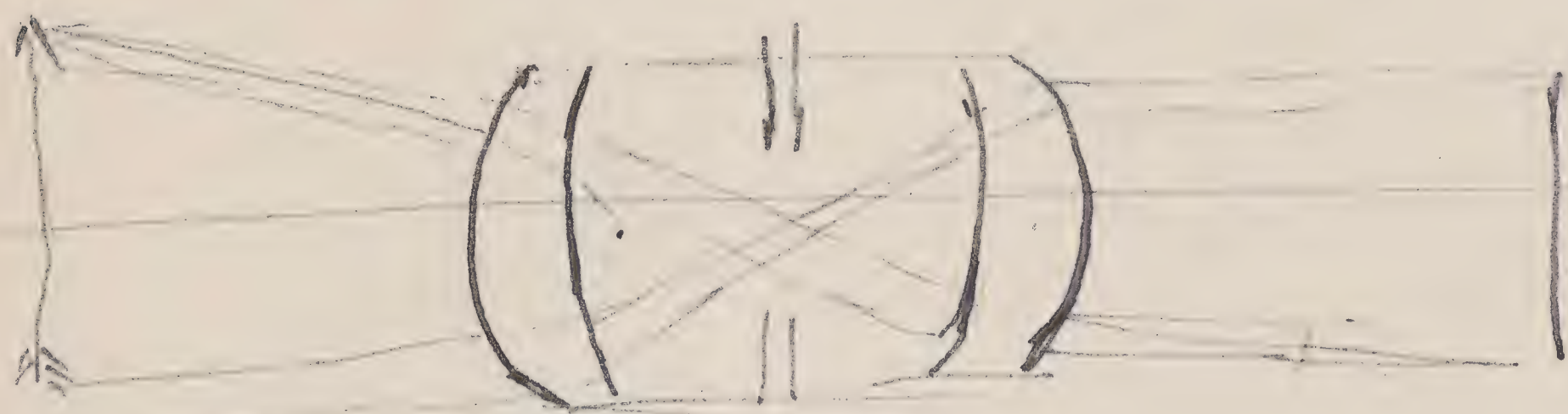
The whole subject of errors in lenses may be treated under three heads.

- 1^o Want of parallelism in the surface
- 2^o Unequal magnification in different parts of the field
- 3^o Spherical aberrations.



If we have a real image
it is magnified more at
the center than the at the
edges.

If the image be virtual
it is magnified more at
the edges.



A reticular combination
or photographic lens
to insure that there is no
spherical aberration in
diaphragm or print.

In order to compare
two lenses of unequal
focal length we must

have the same illumination.
This is determined by the
area of the stop.

r = radius of stop

f = focal length

$\frac{r^2}{f^2}$ must be constant

for all two lenses.

In the case of
microscope objectives the
ordinary lens are insufficient
to account for aberrations.

Smith observed that for
every lens there are two
points for which it is
aplanatic, one slightly
nearer the lens than its
principal focus, the other

a little further away than
the focus.

Taking then a combination
of achromatic lenses.

Put the object in the
mean of the two optical
pts. of the first lens.

Put the second lens so
that the optical image
of the 1st lens coincides
with its optical point.

By arranging the successive
lenses in this manner the
spherical aberration may
be in a large measure
obliterated.

Again by over correcting
the 1st lens and under
correcting the second a balance

may be effected.

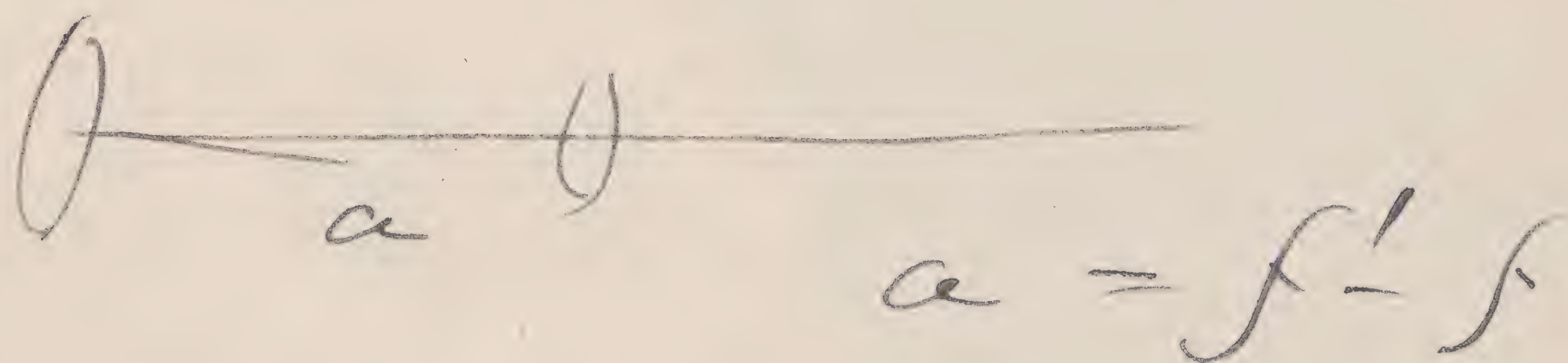
A dry specimen may
have an angular aperture
of 100°

Often the spec is put
into water or oil and
the rays go into the lens
without a sudden change
Immersion the object and
the cover glass in a
homogeneous medium.

Eye-piece

Hughes's eye piece
was planned to get a
good field and decrease
the spherical aberration
by using two weak lenses

instead of one thing
we bend the rays. —



If the two kinds of
lenses are of the same
glass it is impossible
to make the resultant
image the same in
size and position for
all wave lengths. We
can do better of these
things — i.e., we may
make them all
the same size or in the
same position. The
former is preferable. In

order that the images
be the same size

$$a = \frac{L + L'}{2}$$

Both this and the preceding
condition are satisfied
if $3L = L'$ - where
the relation is, however
reversed.

In constructing achromatic lenses, the two spectra are unlike - and it is impossible to make a perfectly ach. lens.

Suppose we have a single lens.

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{r} - \frac{1}{s} \right)$$

for a double lens.

$$\frac{1}{\phi} = \frac{f + f' - c}{ff'}$$

neglecting c .

$$\frac{1}{\phi} = \frac{1}{f} + \frac{1}{f'}$$

what if the case in telescopic objectives - and this becomes.

$$\frac{1}{f} = (\mu_1 - 1) \left(\frac{1}{r} - \frac{1}{s} \right) + (\mu'_1 - 1) \left(\frac{1}{r'} - \frac{1}{s'} \right)$$

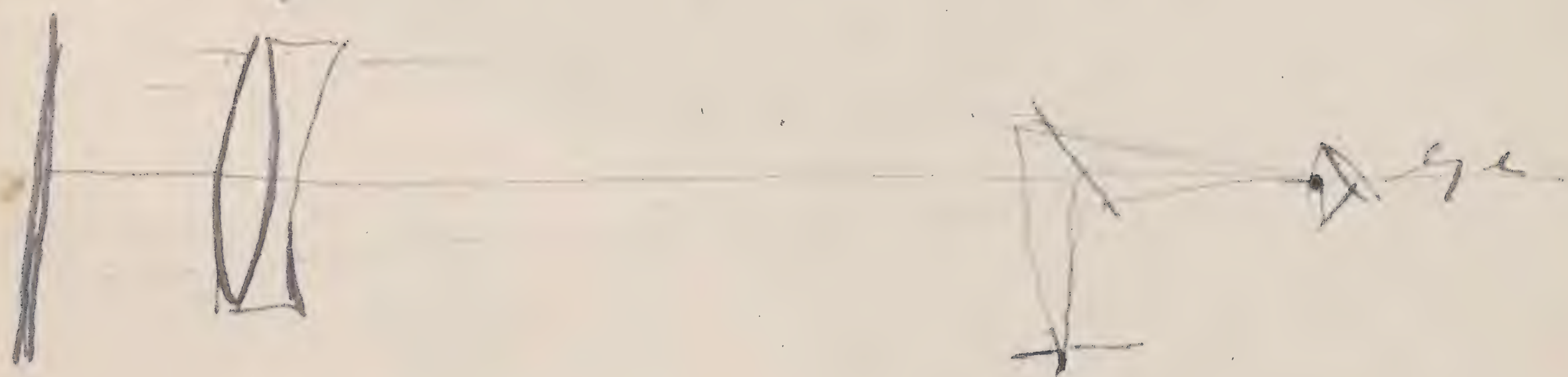
$$\frac{1}{s} = (\mu_1 - 1) \left(\frac{1}{r} - \frac{1}{s} \right) + (\mu'_1 - 1) \left(\frac{1}{r'} - \frac{1}{s'} \right)$$

i.e. making the focal length
the same for a wave length
in the red and another in
the green, as we would for
spectacles - for a
photographic objective
would correct for rays
in white light -

Have four ability centers
and but two conditions:
the third surface, is
used to correct spherical
aberration - the front
surfaces, made to coincide
with second to prevent
reflection and loss of light -

The last for
spherical aberration -
The lens is set up in front

of a plane mirror of the
size of the lens.



the point placed in
prime. focus - reflected
back and comes to a
focus at the eye - Every
pt. of the lens should appear
equally illuminated -
Extremely delicate test.

Spectroscope

Newton - circular hole -
O (O)

Hollander 1802 used slit.



1814 Fraunhofer added
the telescope - discovered
the lines. - But the
not at a great distance
in order, to get more view
in vision, except!

1834 Luminous introduced
the collimating lens

forming a flat wave
falling on the surface

of the mirror, a flat surface,
will be refracted as a
flat wave - hence are
no longer subjected to
position of minimum
deviation to get good
definition - But
light of one wave length
may come out as a beam,
but other wave length
may come out as spherical
and must be sure that
of minimum deviation.

Interference

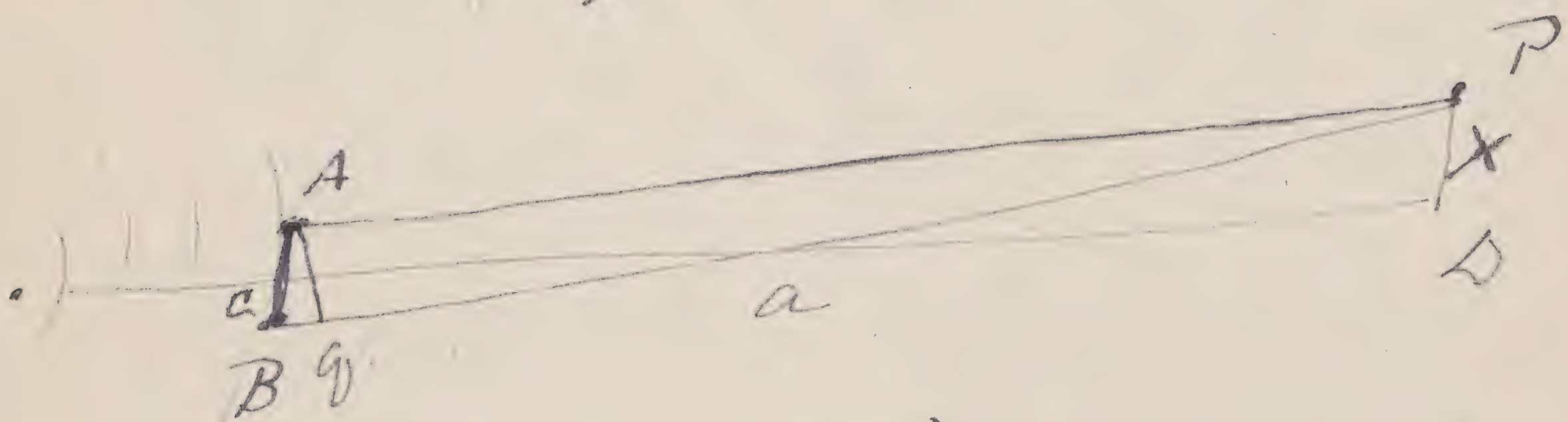


Not complete interference



In order that waves coming from two points should interfere,

Must have two slits, A, B, and the light from B must be part of the same wave front from A.



BQ is the difference in path of AP and BP.

$$\frac{x}{a} = \frac{BQ}{c}$$

$$x = \frac{a}{c} \overline{BQ}$$

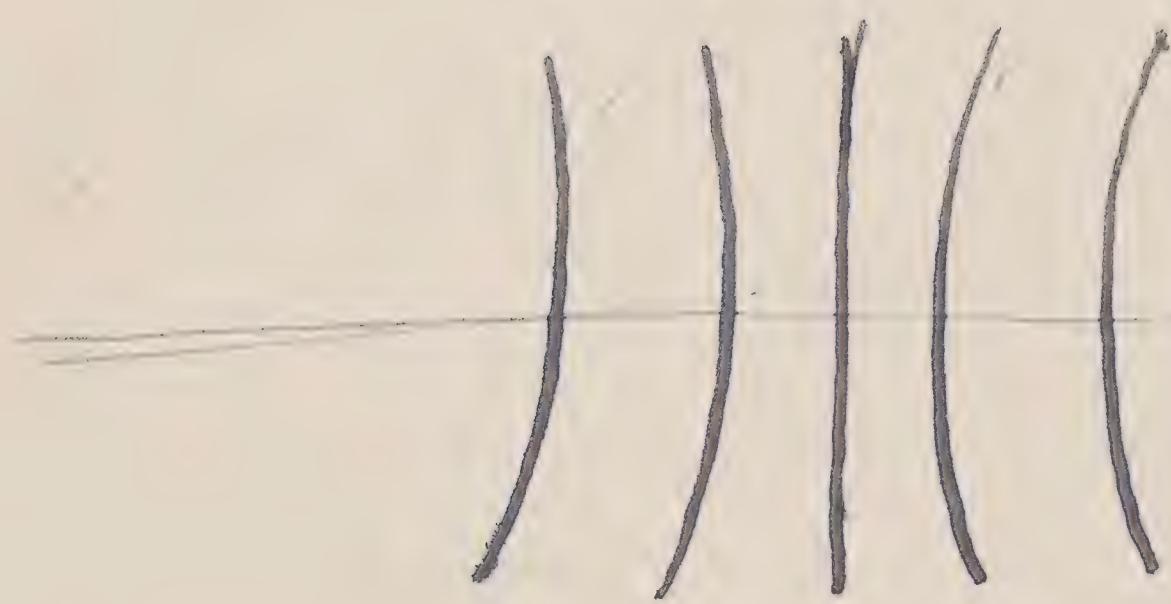
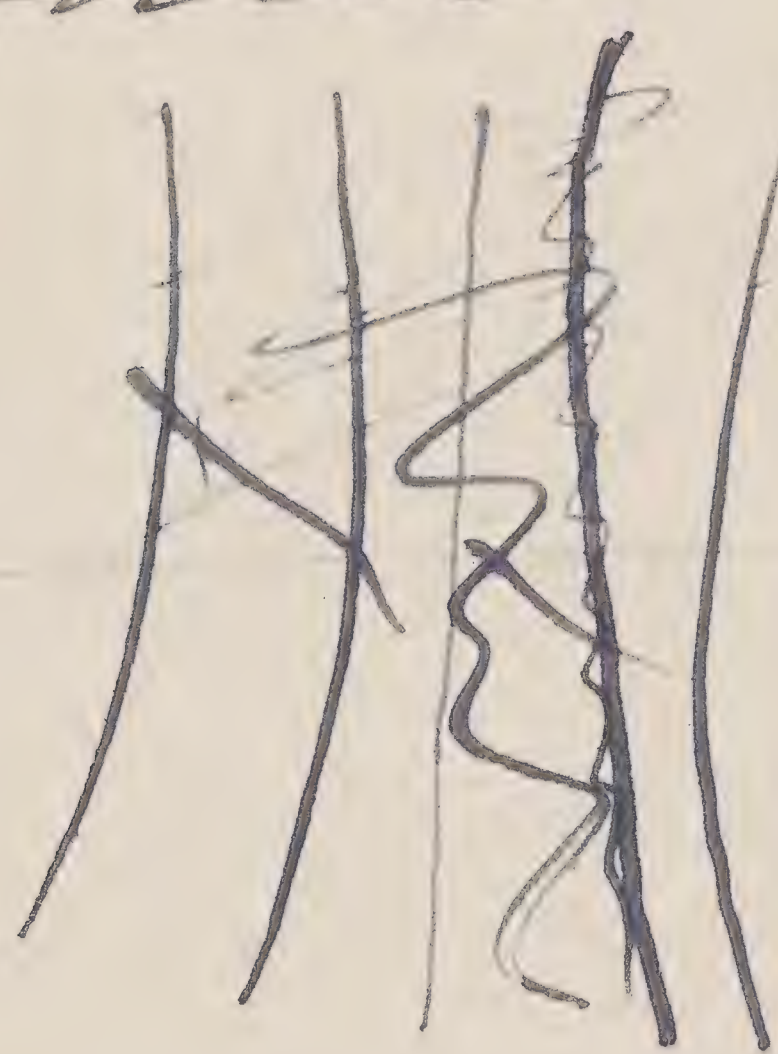
Will have interference when
 B_g is an odd multiple of
 $\frac{1}{2}\lambda$.

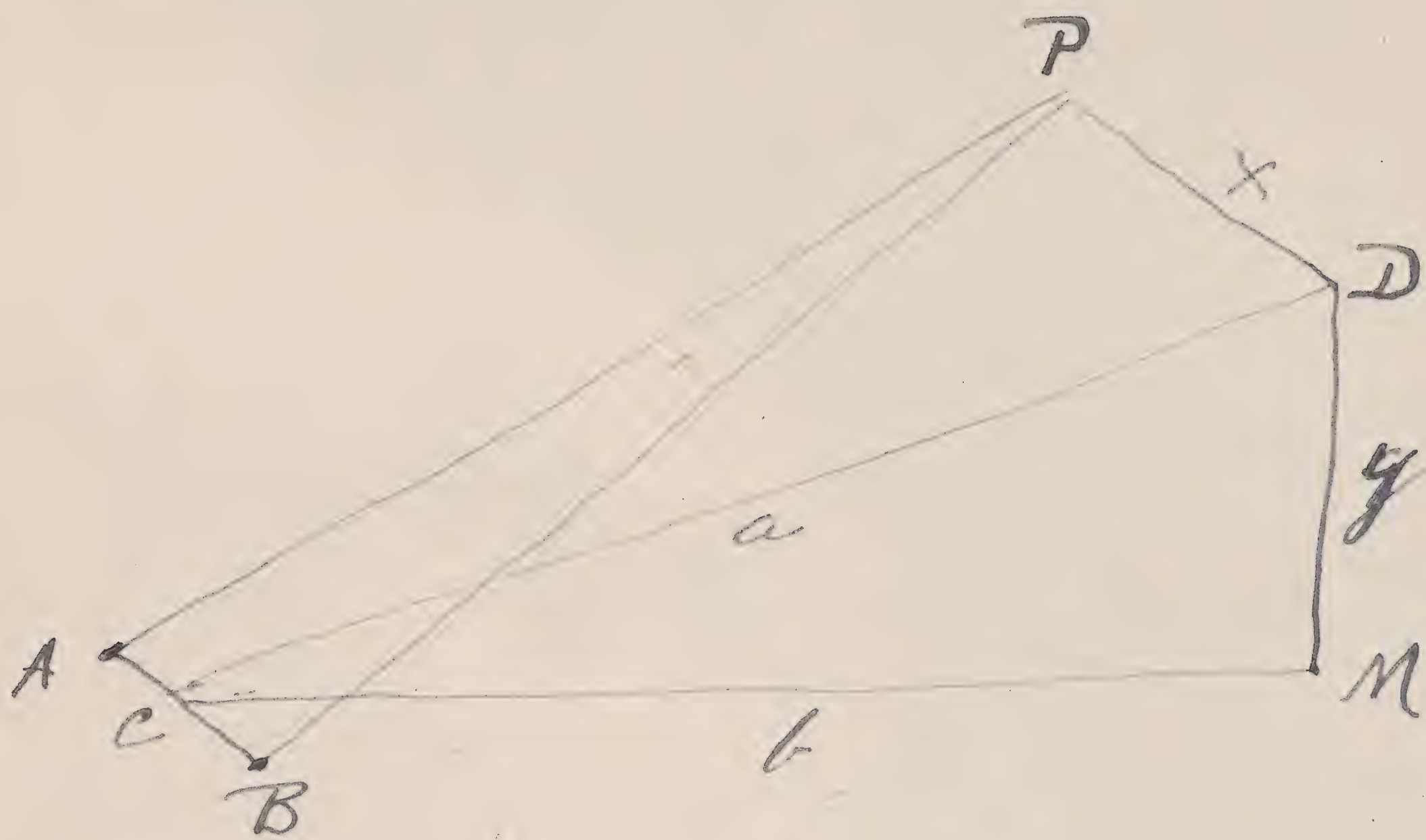
$$x = \frac{a}{c} \cdot \frac{s\lambda}{2}$$

$s = \text{odd integers} = \text{darkness}$

$s = \text{even " or } 0 = \text{brightness}$

The bands will be
of the form



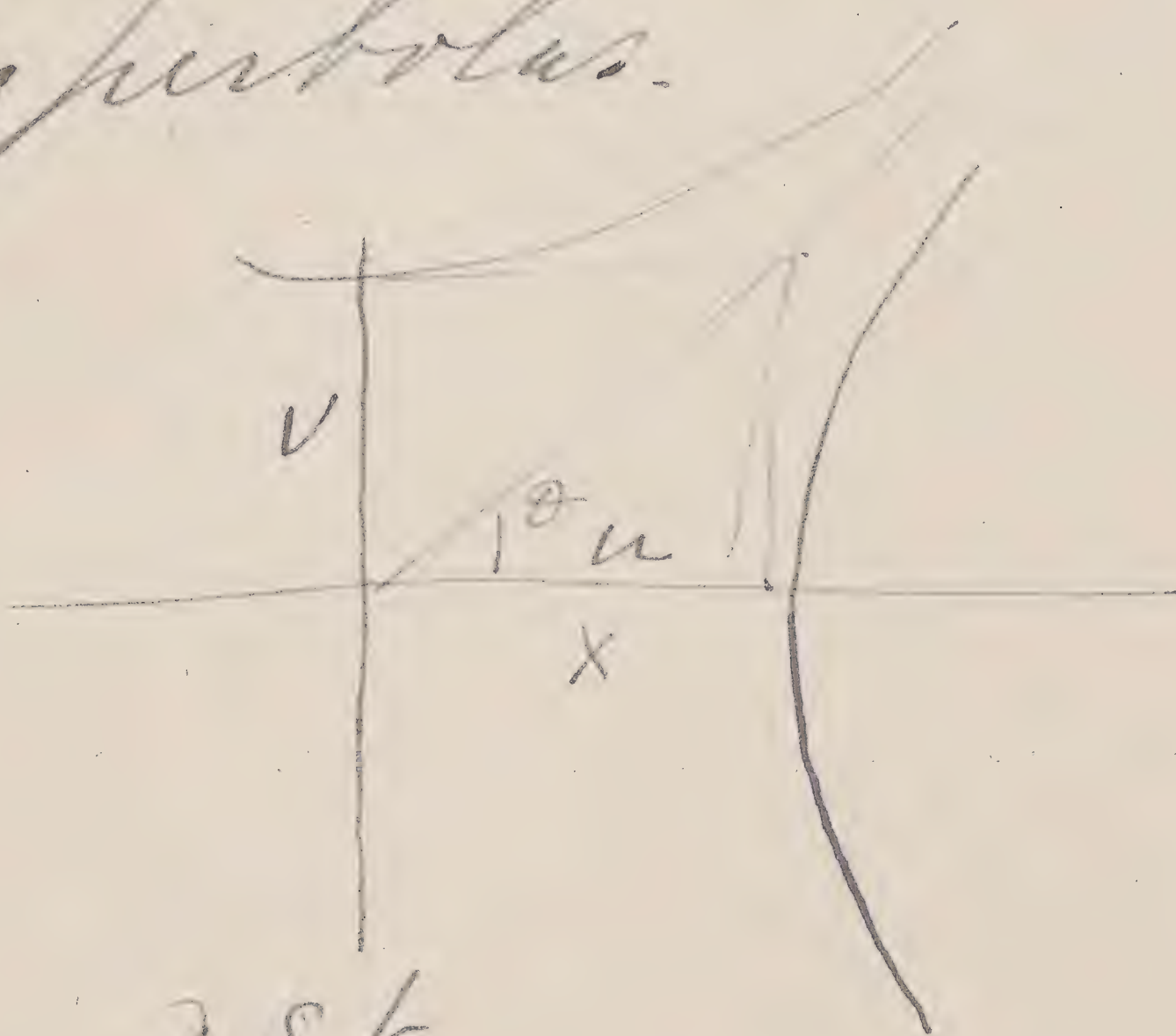


$$x = \frac{a \lambda s}{2c}$$

$$a^2 = b^2 + g^2$$

$$\frac{4x^2c^2}{1^2s^2} - y^2 = 6^2$$

And we find the curves
to be hyperbolas.



$$u = \frac{\lambda s h}{2c}$$

$$v = f$$

$$\tan \theta = \frac{2c}{2s}$$

the byperbolas are almost
straight lines.

Grimaldi - let some
light pass thru' two openings
and found interference

Tho. Young found that
both slits must be
illumined by same
source - He thought it
confirmed the wave
theory - But at that
time Newton, & others, explained
it by diffraction. To
get over this difficulty -
Fresnel devised the
experiment having only
one opening.

The ray after passing one
slit, was reflected from
the mirror.

S_1

S_2

He also devised the
use of the biprism to
replace the mirror.

Brass prepared as a
test for the wave theory
of light, to place in the
path of one ray a piece
of glass.

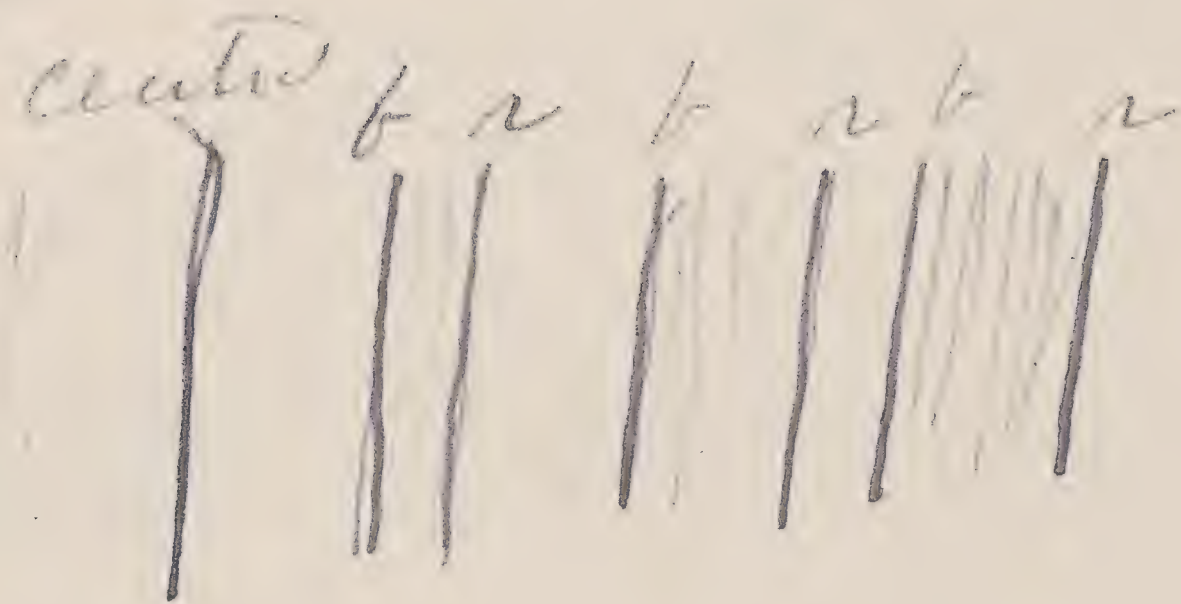


The experiment shows
was to assume that
light travels faster in
glass than air. The
wave theory that the
wave was retarded in the
denser medium.

The displacement of
the light center spot
is always towards the glass.

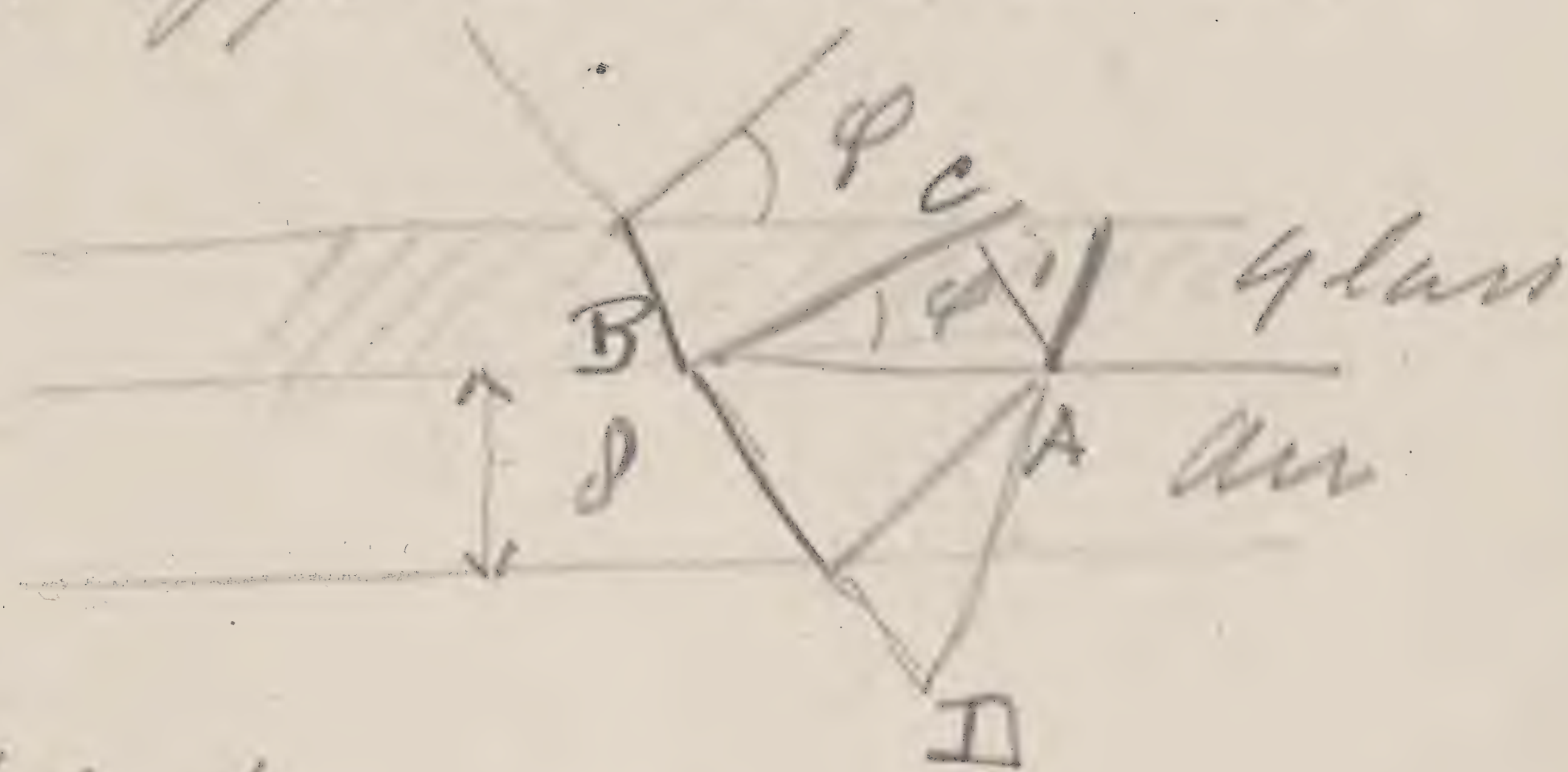
compare the two things.

If we use white
light we get a bright
central line and spectra
instead of side bands.



Newton's rings

Why should a thin film appear colored?



$$t = t' = nT$$

We would not expect to find interference at A

$$t = \frac{BD}{v}$$

$$AB = 2d \tan \phi$$

$$BD = 2d \tan \phi \sin \phi$$

$$t = \frac{2d \tan \phi \sin \phi}{v} = \frac{2d}{v} \cdot \frac{\sin^2 \phi}{\cos \phi}$$

$$\text{Path of part } CA = BD$$

$$\text{Path } B \epsilon A = \frac{2d}{\cos \phi}$$

$$t' = \frac{2d}{\cos \phi v}$$

time on path $B \epsilon - \epsilon A$

$$t' - t = \frac{2d}{v} \left(\frac{1 - \sin^2 \phi}{\cos \phi} \right)$$

$$= \frac{2d}{v} \cos \phi = nT = \frac{\lambda}{v} n$$

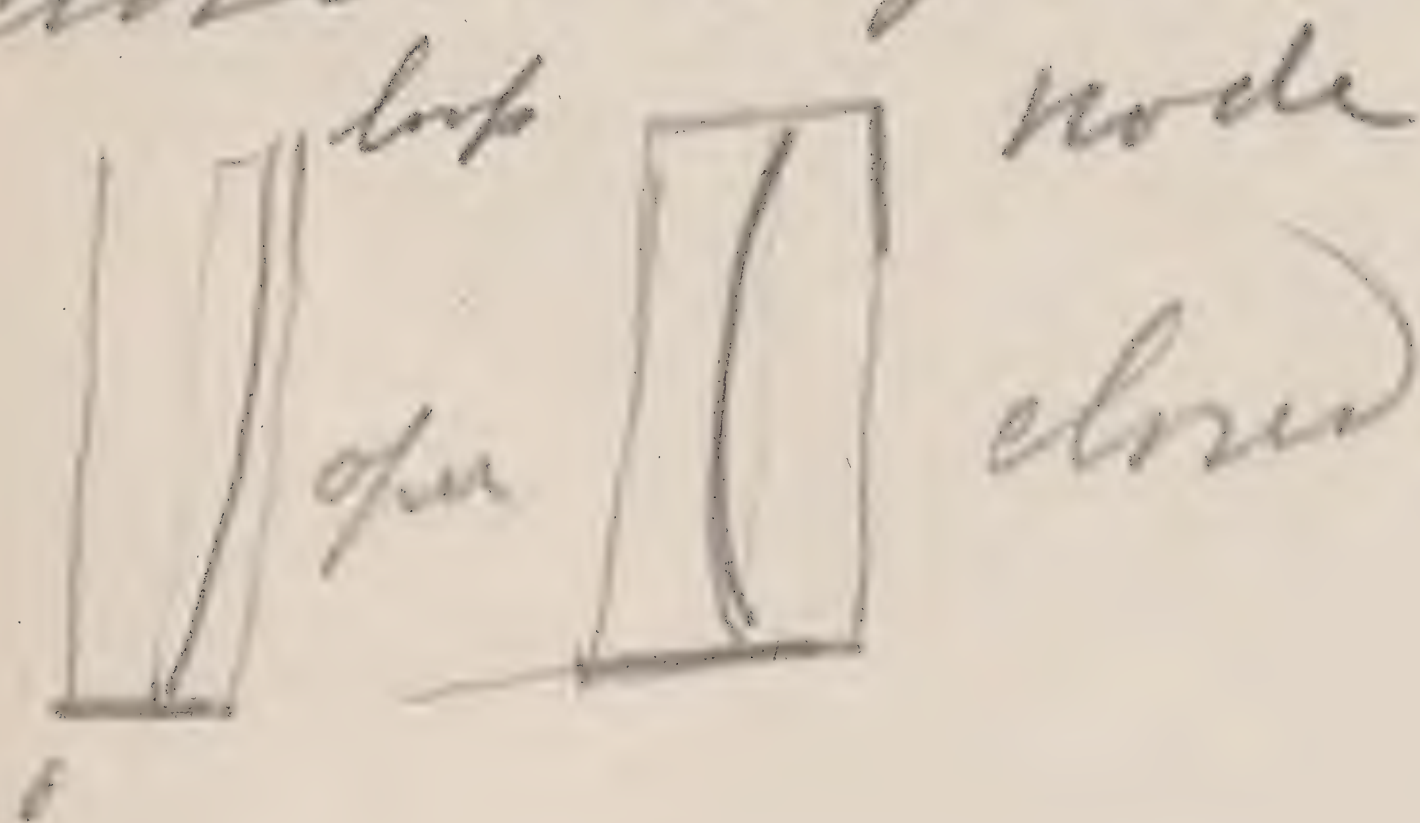
$$d = \frac{n\lambda}{2 \cos \phi}$$

this corresponds to dark bands
altho' the difference in
path is a whole number
of wave lengths

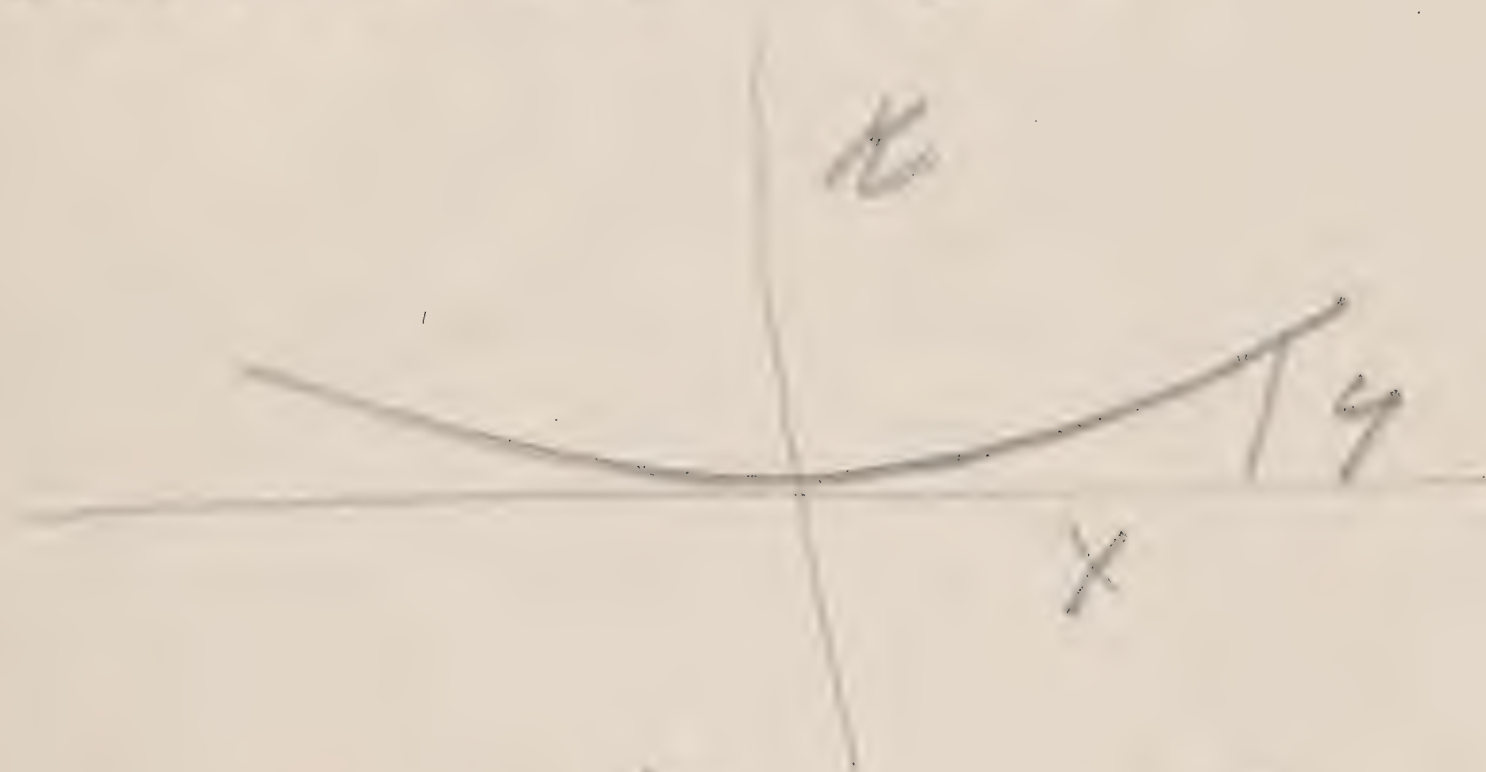
The reflection at the
different surfaces cause
this. In one case the
reflection is at a more
dense medium, in the

other at a less dense -

Compare open and
closed organ pipes



Newton's rings



$$y = \frac{x^2}{2R} = \frac{n\lambda}{2}$$

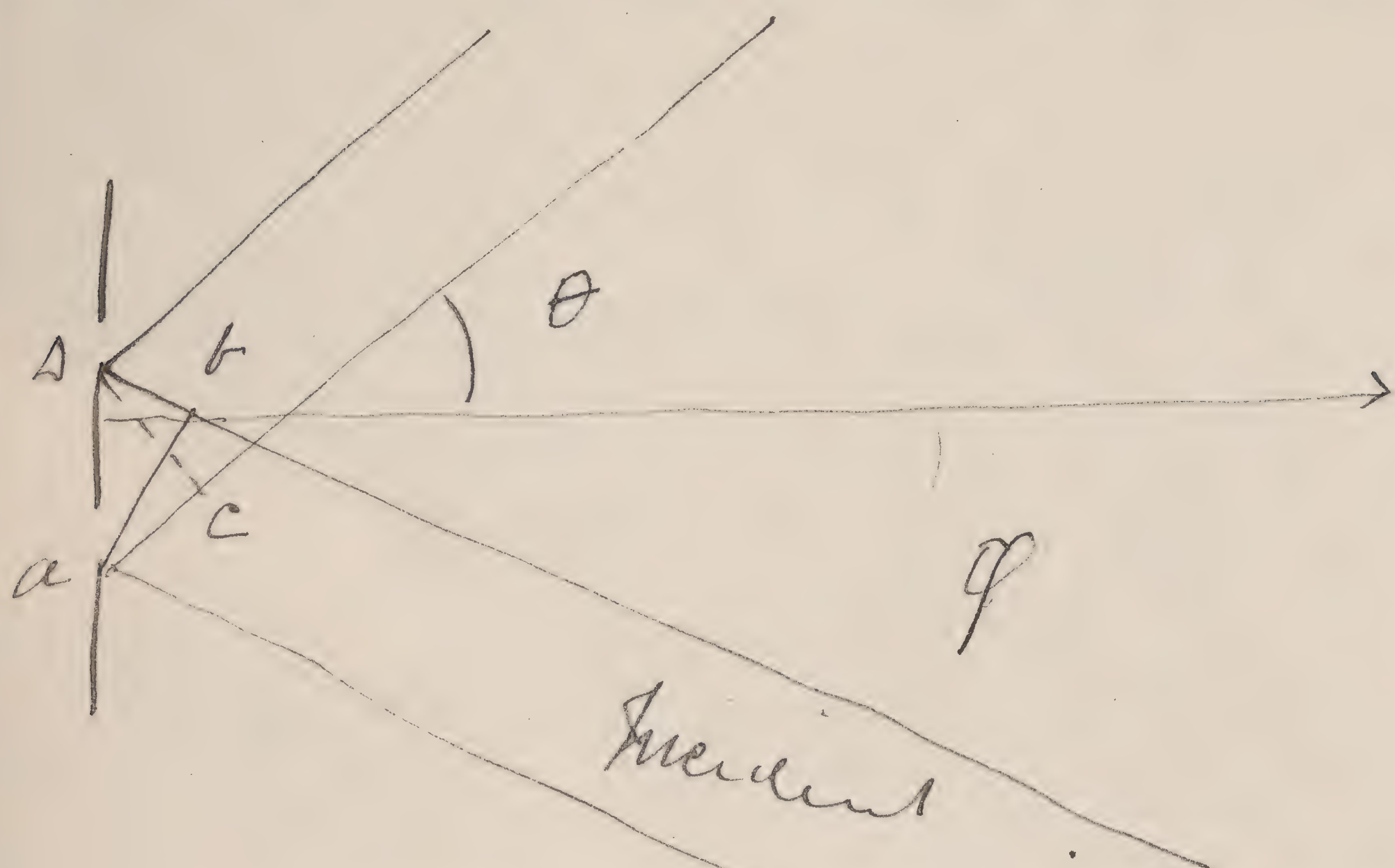
$$x^2 = \frac{n\lambda}{R} \quad \text{dark}$$

When

$$x^2 = 2n \left(\frac{\lambda R}{2} \right) \quad \text{dark}$$

$$x^2 = (2n+1) \left(\frac{\lambda R}{2} \right) \quad \text{bright.}$$

Milne's Gating.



$ac - bd =$ difference in path of the two waves. and this must for a bright line be equal to a whole number of wave lengths

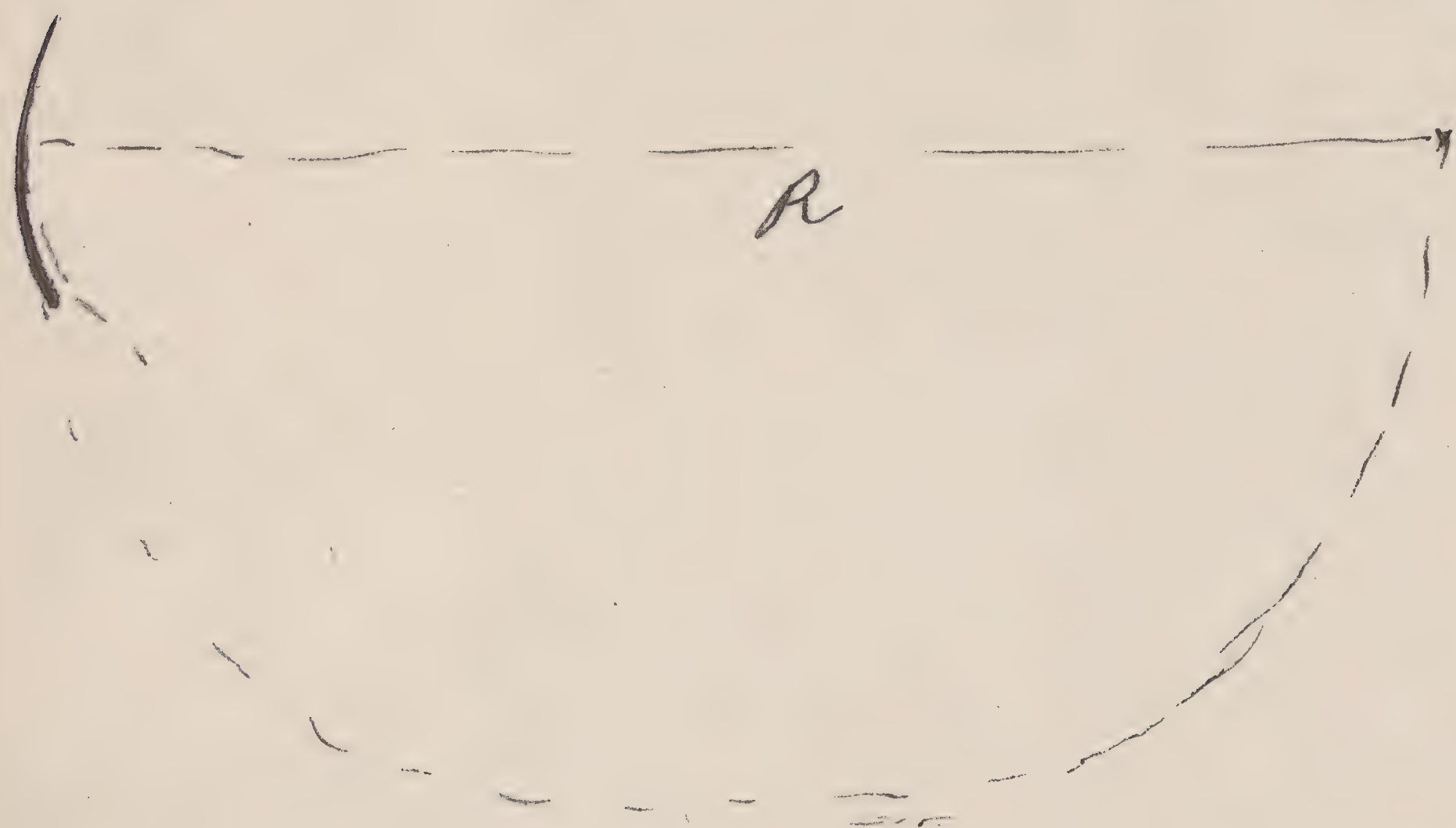
$$s \sin \theta - s \sin \phi = n \lambda$$

$$\lambda = \frac{s}{n} (\sin \theta - \sin \phi)$$

We will have total interference if the difference in path is any thing but a whole number of wave lengths, in account of great

number of lines.

Curved Grating

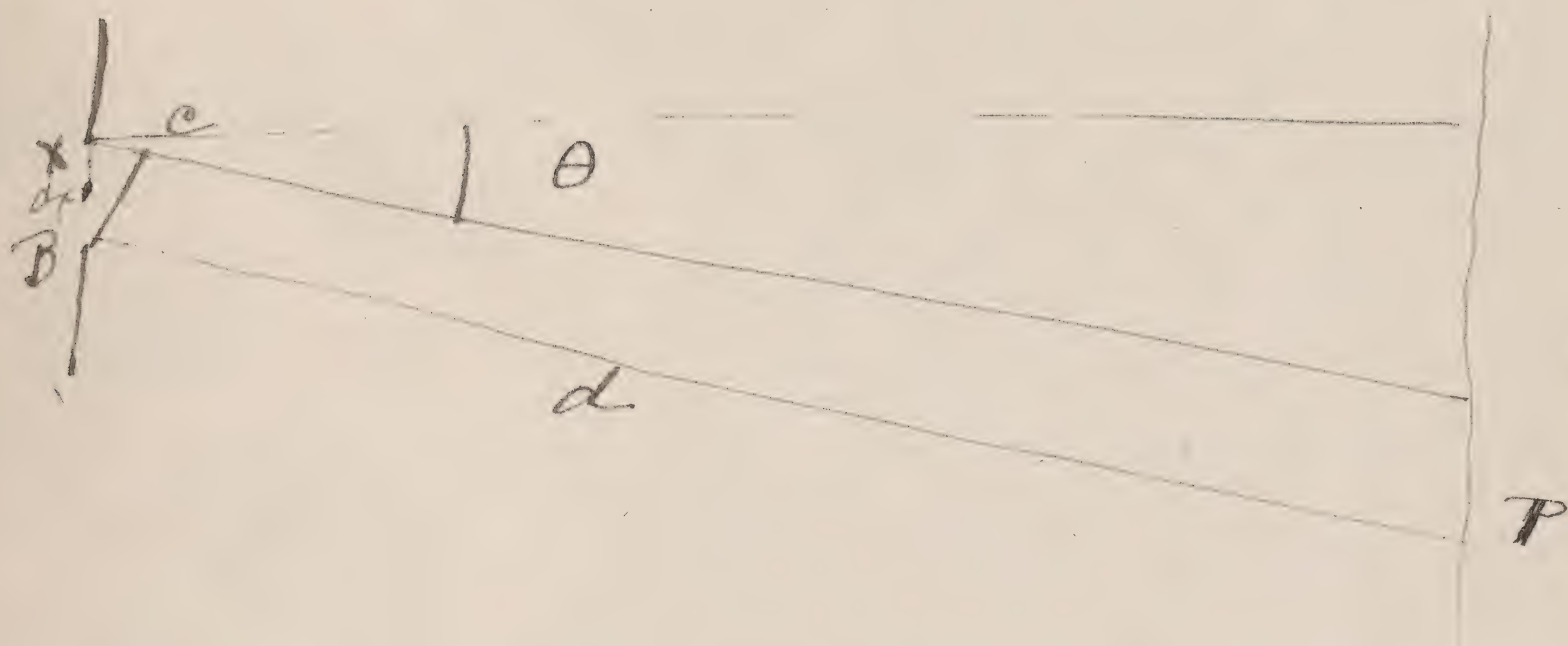


If R be the radius of curvature of the grating, and if we draw a circle with R as the diameter; then if the slit be on this circle, the various spectra will be in focus and (normal) on the circle.

It is found that the
spectrum directly opposite
the grating is normal.

$$W = \int \frac{p}{a^4} \sin \frac{2\pi}{\lambda} (vt - d - x \sin \theta)$$

p = amplitude at the slit.



The above expression is the displacement of a particle at P due to the amplitude p at the slit.

The only part that varies with x is the part under the (): put this

$$\frac{2\pi}{\lambda} (vt - d) = \phi.$$

$$\frac{2\pi}{\lambda} x \sin \theta = \psi.$$

and we have

$$N = \frac{L}{a^2} \int \sin(\varphi - \psi) dx$$

$$= \frac{L}{a^2} \sin \varphi \int \cos \psi dx$$

$$- \frac{L}{a^2} \cos \varphi \int \sin \psi dx$$

What we want is the square of the amplitude - The amplitudes of the components are.

$$\frac{L}{a^2} \int \cos \psi dx; \quad \frac{L}{a^2} \int \sin \psi dx$$

We have the formula.

$$y = a \sin \varphi + b \cos \varphi$$

$$= A \sin(\varphi + \chi)$$

$$A^2 = a^2 + b^2, \quad \tan \chi = \frac{b}{a}$$

where A^2 is the square of the amplitude of the resultant disturbance.

We have therefore

$$I = A^2 = \left(\frac{L}{a^2} \int \cos \psi dx \right)^2 + \left(\frac{L}{a^2} \int \sin \psi dx \right)^2$$

Replacing x by its value and
integrating.

$$\left[\frac{\lambda}{2\pi \sin \theta} \sin \frac{2\pi x \sin \theta}{\lambda} \right]_0^{2b}$$

and by integrating the second
term

$$\left[\frac{\lambda}{2\pi \sin \theta} \cos \frac{2\pi x \sin \theta}{\lambda} \right]_0^{2b}$$

We must now square
and add.

$$\frac{\lambda^2}{d^2} \frac{\lambda^2}{4\pi^2 \sin^2 \theta} \left[\sin^2 K + \cos^2 K - 2 \cos K + 1 \right]$$

$$= \frac{\lambda^2 \lambda^2}{d^2 2\pi^2 \sin^2 \theta} (1 - \cos K)$$

$$= \frac{\lambda^2 \lambda^2}{d^2 2\pi^2 \sin^2 \theta} \cdot 2 \sin^2 \frac{K}{2}$$

and the result is

$$I = \frac{I^2 \lambda^2}{d^2 \sin^2 \theta} \left(\sin \frac{2\pi b \sin \theta}{\lambda} \right)^2$$

$$= \frac{4 I^2 b^2}{d^2} \frac{\left(\sin \frac{2\pi b \sin \theta}{\lambda} \right)^2}{\left(\frac{2\pi b \sin \theta}{\lambda} \right)^2}$$

$$= \frac{4 I^2 b^2}{d^2} \left(\frac{\sin \phi}{\phi} \right)^2$$

Minimum points are

$$\sin \phi = 0 \quad \text{or} \quad \frac{2\pi b \sin \theta}{\lambda} = n\pi$$

$$\sin \theta = \frac{n\lambda}{2b}$$

This gives dark bands.

Maximum points

$$\text{are } \left(\frac{\sin \phi}{\phi} \right)^2 \text{ a max.}$$

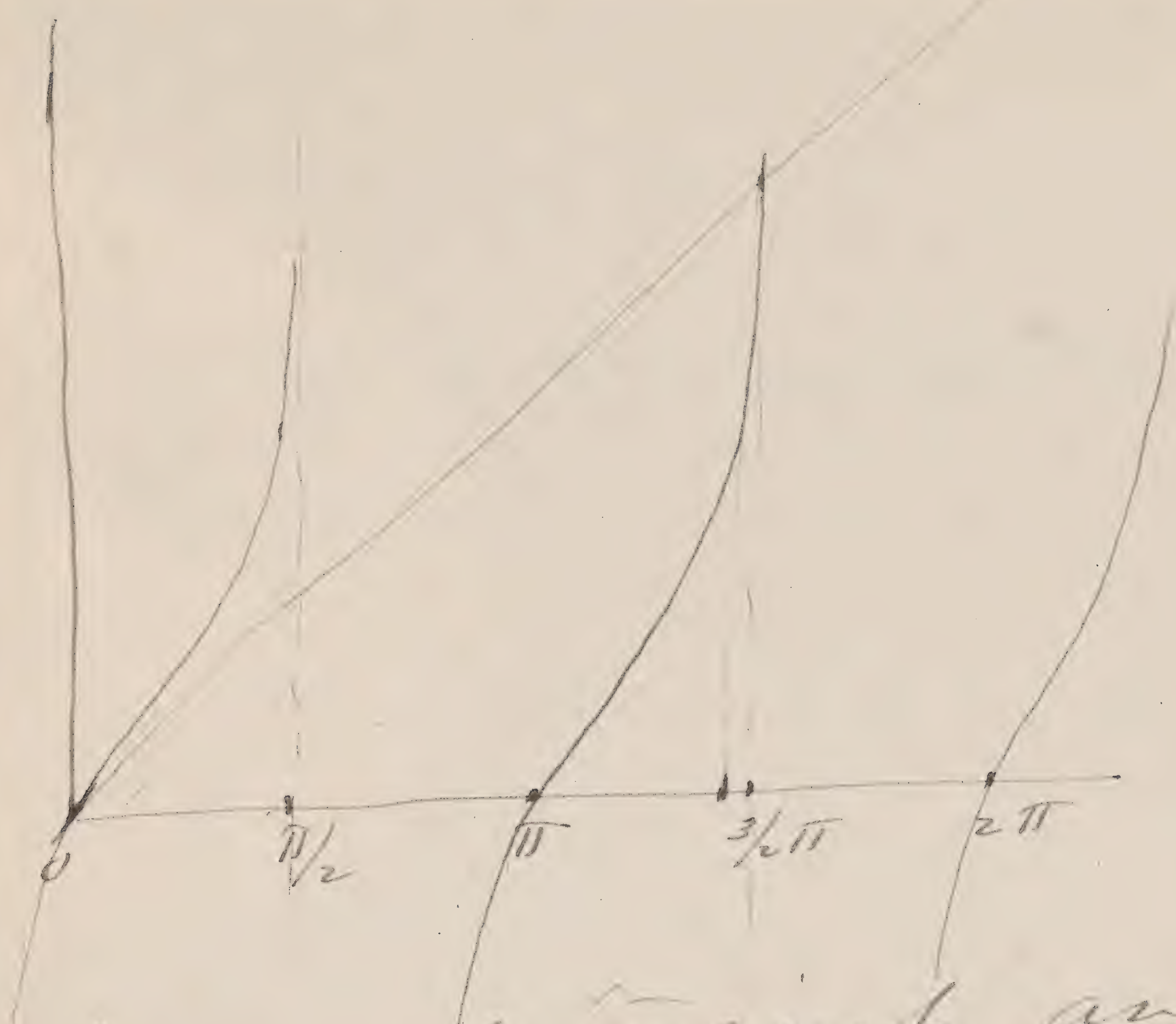
that is when $\tan \phi = \phi$.

$$\phi = 0$$

$$\phi_1 = 2.86 \pi/2$$

$$\phi_2 = 4.92 \pi/2$$

$$\phi_3 = 6.94 \pi/2$$



Curves of argument, arg. to
 indicates that $\pi/2, 3\pi/2$ etc
 line $\phi = \tan \phi$, includes 45° .

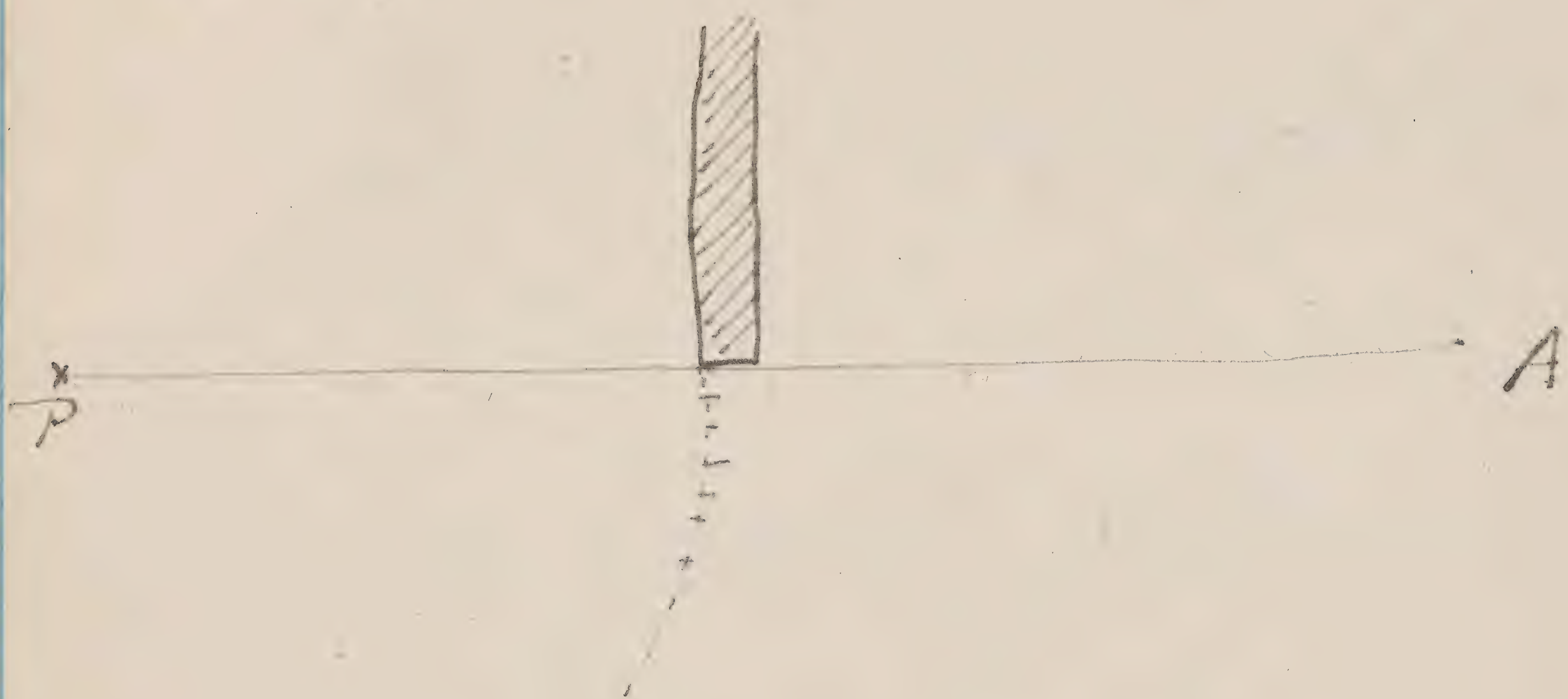
The law of dissemination
 of intensity is

$$1, \frac{4}{9\pi^2}, \frac{4}{25\pi^2}, \text{ etc}$$

The light lines diminish
 in intensity very rapidly.

of small first investigated
diffraction phenomenon -

Diffraction around the
edge of screen -

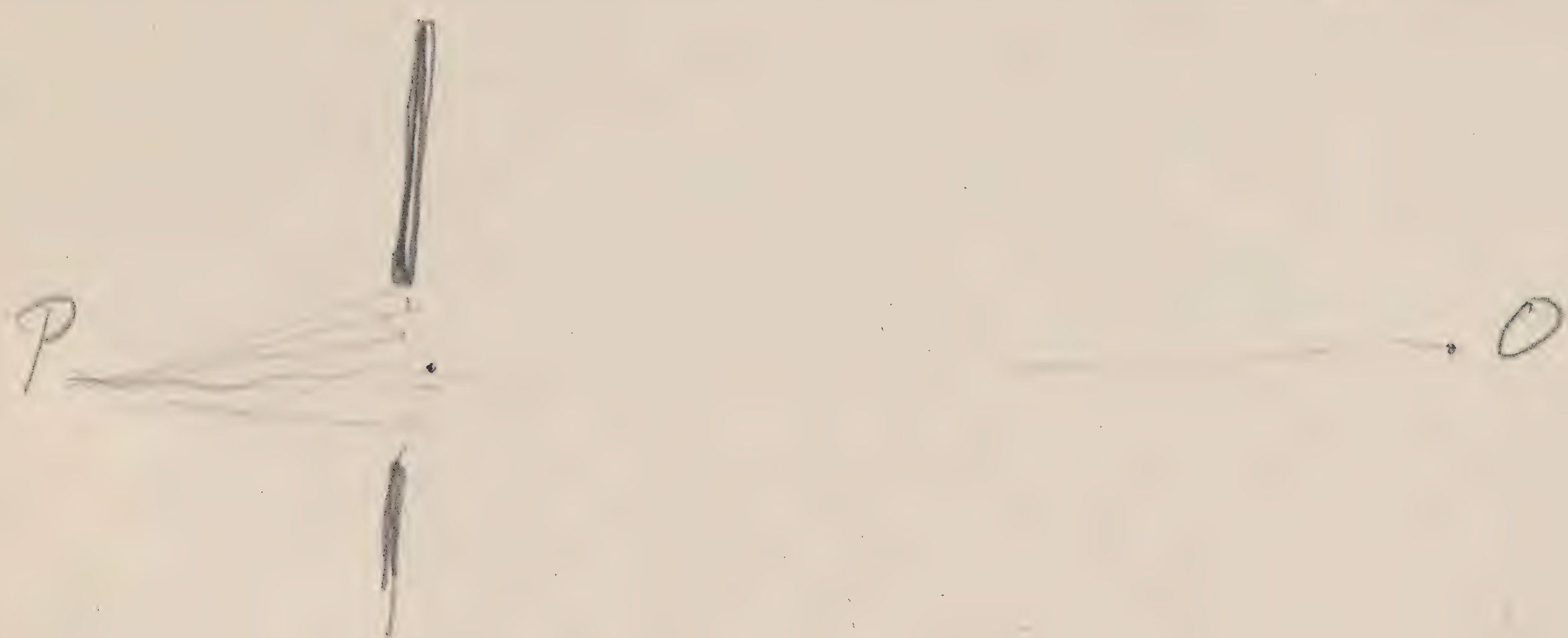


The point A on the edge
of the shadow is a point
of half the illumination - as when
there is no screen -

If the screen were drawn
back a half wave length



P will be a maximum
except when it is a min.
and we have diffraction
fringes around P.



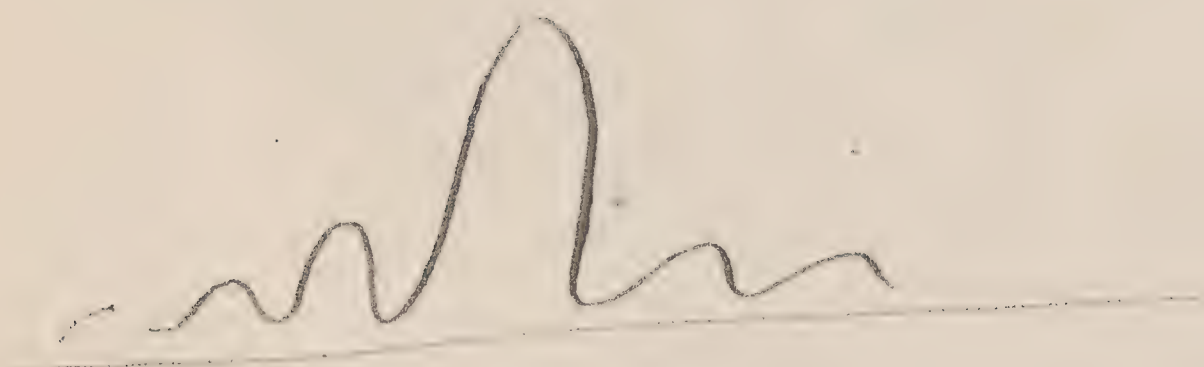
The point P may be light
or dark - Divide the
opening up into half period
circles - The areas of all
the rings are equal

$$r_1^2 = \frac{abx}{a+b}$$

$$r_n^2 = \frac{2abx}{a+b}$$

If we take the opening
such that an even number
of rings are including, the
point P would be dark
If an odd no. light

By cutting out every other
line, the point P would
be more than 1/2 illuminated



The image will be surrounded by bands.
Fraunhofer diffraction thru
lenses—

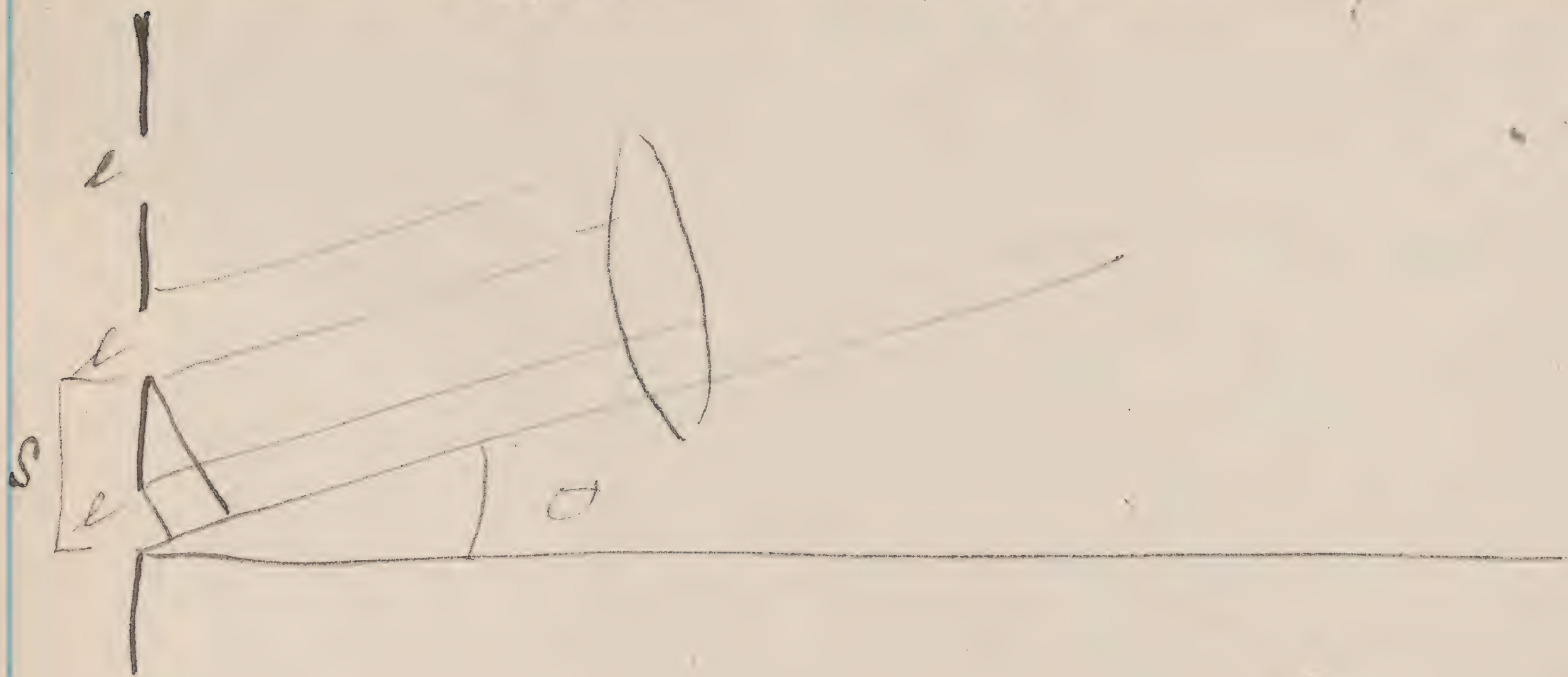
$$\frac{R\theta}{\lambda} = 0.61 \text{ for first dark ring}$$

$$\left. \begin{array}{l} R = 10 \text{ cm} \\ \lambda = .00006 \end{array} \right\} \theta = \frac{7}{10} \text{ arc} -$$

This limits the resolving
power of telescope—

$\theta = \frac{2}{3}''$ could be resolved
let more than $\frac{1}{2}$ the
central spot is vis—

Diffraction Grating



There are diffraction effects depending upon each opening separately, and upon the combined.

$$\sin \theta = \frac{n \lambda}{e} \quad \text{darkness}$$

$$\sin \theta = \frac{2n+1}{2e} \lambda \quad \text{brightness}$$

The eye telescope combines all the rays, and brings them to one focus. Intenseifies the diffraction effect due to each opening separately.

$$\frac{7}{8} S \sin \theta = \frac{2n+1}{2} \lambda \text{ dark}$$

$$S \sin \theta = \frac{n \lambda}{S} \text{ bright}$$

this is the effect due to
two openings -

Ramton

The elementary theory
first proposed by Des cartes -
Hollies that light reflected
by sphere at minimum
incidence has minimum
light.

Exact measurements
of the angles of deviation
etc did not correspond
to the theory.

Young first gave
the theory of interference
as the cause - Young
completed the theory.

If we trace the
wave front as it
passes thro' the slit

It goes in flat but
comes out as a surface
with a point of reflection



At the point of reflection
the rays come out // and
with minimum deviation

The point of maximum
illum. is a little above
the point given by
Descartes theory. Above
this point Ray forms
a series of max. and
min.

Then 2nd and 3rd max
give other rows under
the primary. - Then

could not be accounted
for by Descartes theory

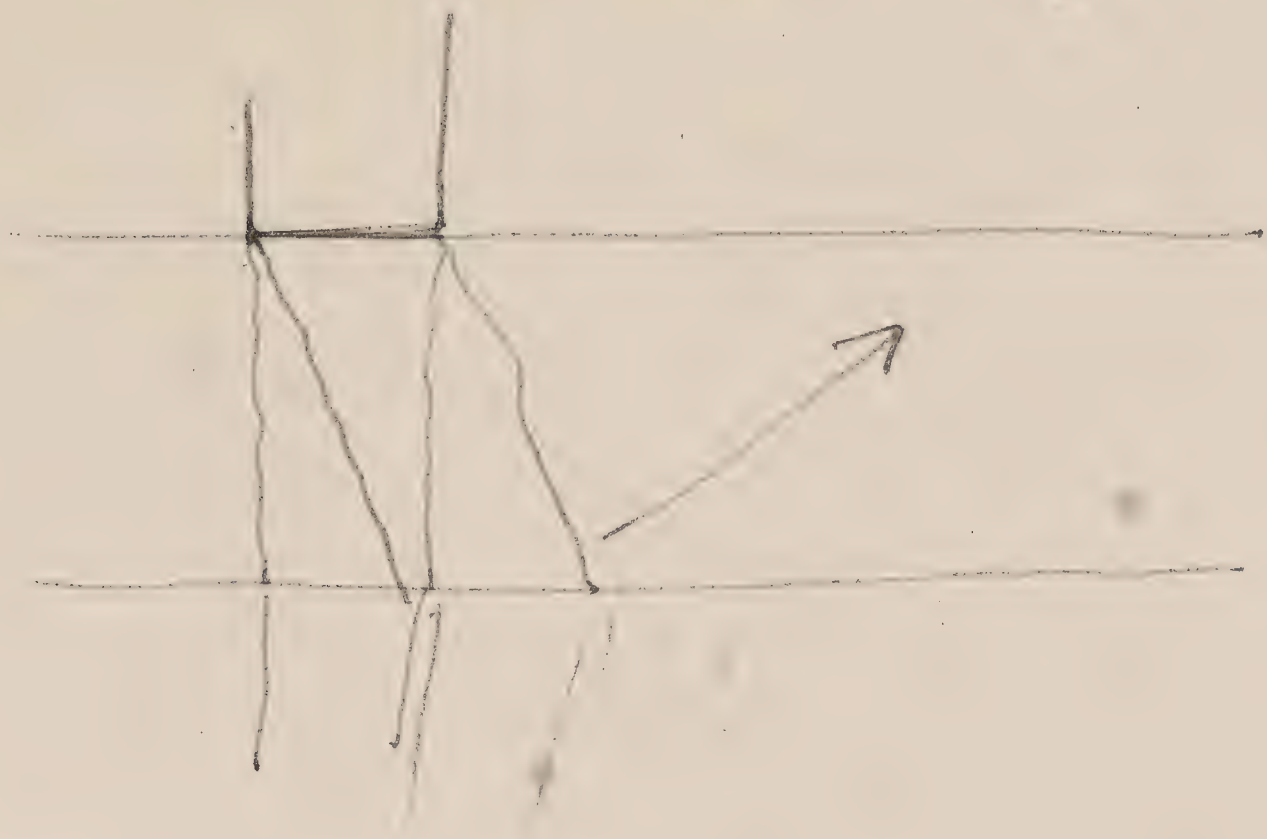
Double Refraction

There are in the crystals three principal axes of elasticity. One of them coincide with the crystall. axis; the other two at rt. angles to this. The elastic properties of the crystal are symmetrical around the crystall. axis.

The opt. or crystal. axis is ^{an axis} ~~a plane~~ of symmetry and a plane thro' it a principal plane.

When a beam of light passes into spar is split up into two beams; one

pares this as if thro' glass.



One ray always obeys
the ordinary law of refraction.
The other does not.

The velocity of light
in the medium is not
a constant; varies with
the inclination to the
optic axis.

The extraordinary ray
in a plane perpendicular
to the optic axis obeys the
law of refraction. But

its index of refraction is
diff. from that of ordinary
ray -

We shall follow
Fermat.

Assumed that vibrations
take place in a direction
"to principal plane and
in other ray at right
angles to this plane -

The ordinary ray
is said to be plane
polar. in the principal
plane - the extraordinary
in a plane at right angles
to it.

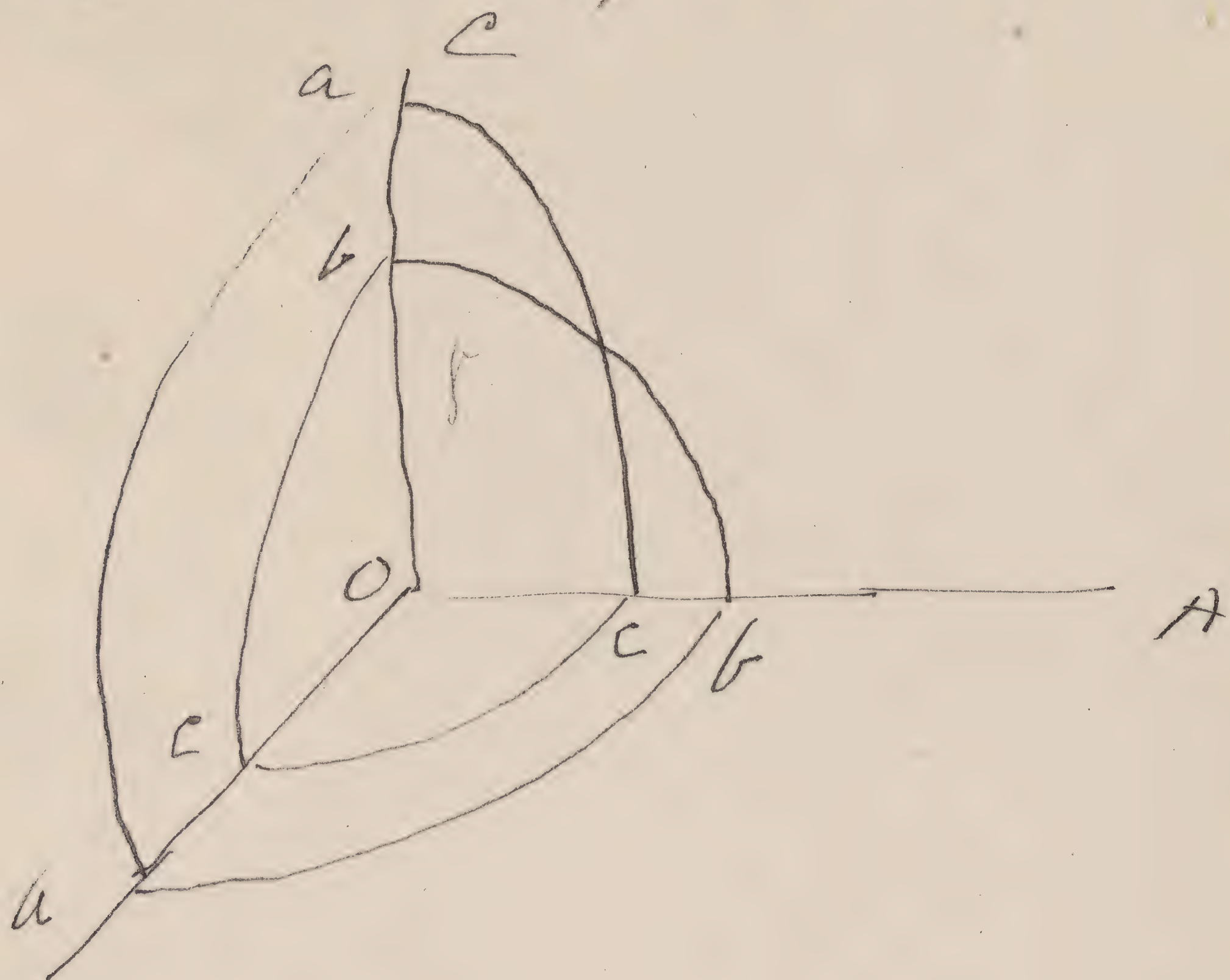
It is a question whether
the vibration takes place
" or $\perp$ to the plane of
polarization -

1 Assume - that in
these rays the light is
plane polar. the this
vibrates in a straight
line.

2 The direction of
the osc. is at 90° angles
to the plane of polar.

3 Elasticity of displacement
is different in one direction
from another -

This is the fundamental
assumption of Fresnel; it
is not perfectly true. Reflection
would lead us to the conclusion
that the density of ether
was different: exactly
the same. Our theory
won't explain reflection,
the other double refraction.
The Electro-magnetic
theory explains both -
I shall however use
Fresnel's -

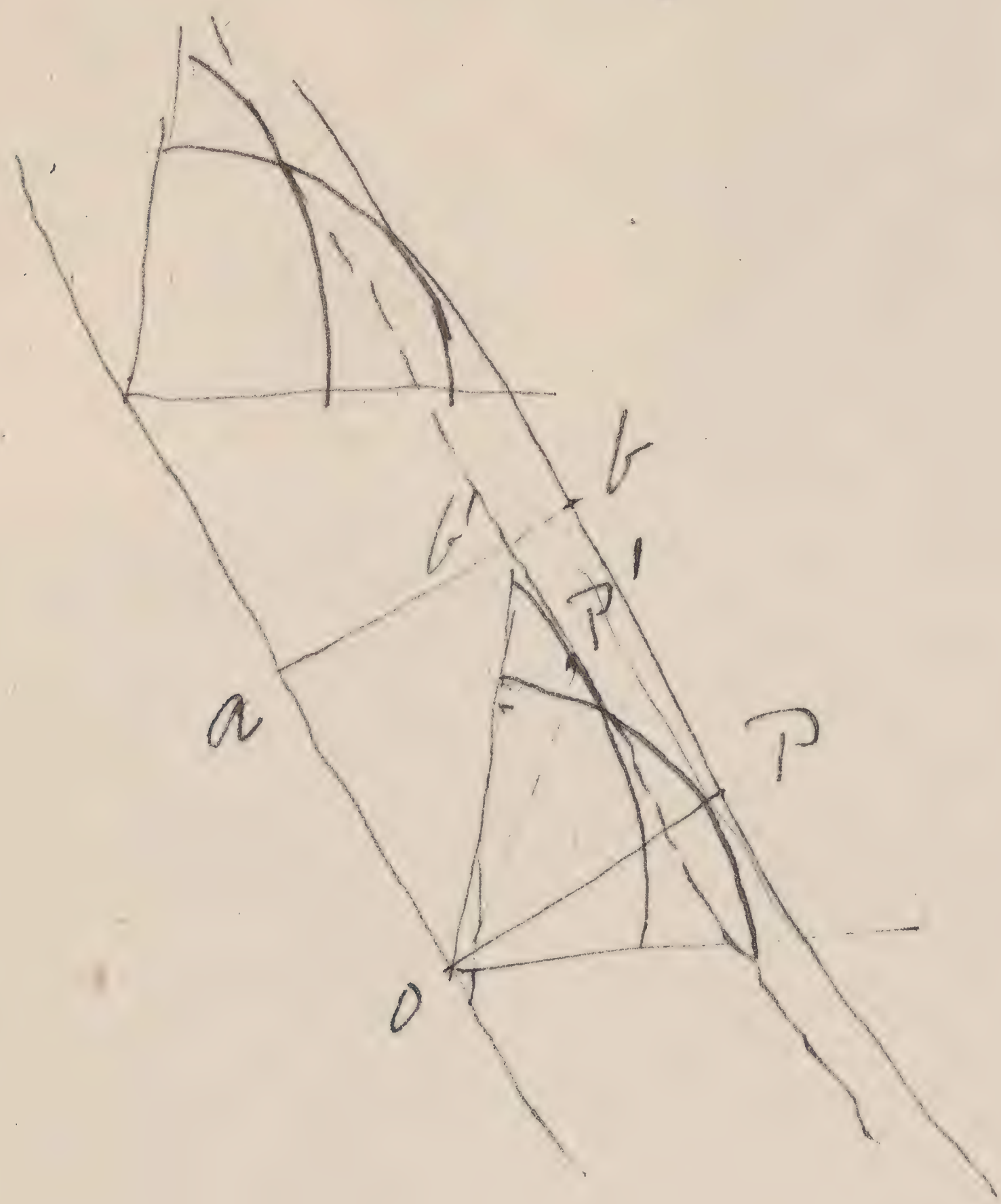


v 3

In the circ. pt. the vibrations at $\perp$ to the plane COH, in the ellipse ac, in the plane COA - And similarly for the other planes.

In the above the declination
comes from the point
O.

To find the form of the
wave due to a plane
wave



We will have two wave
fronts, parallel to the original
proceeding with different
velocities, and the
vibrations taking place
in different planes.

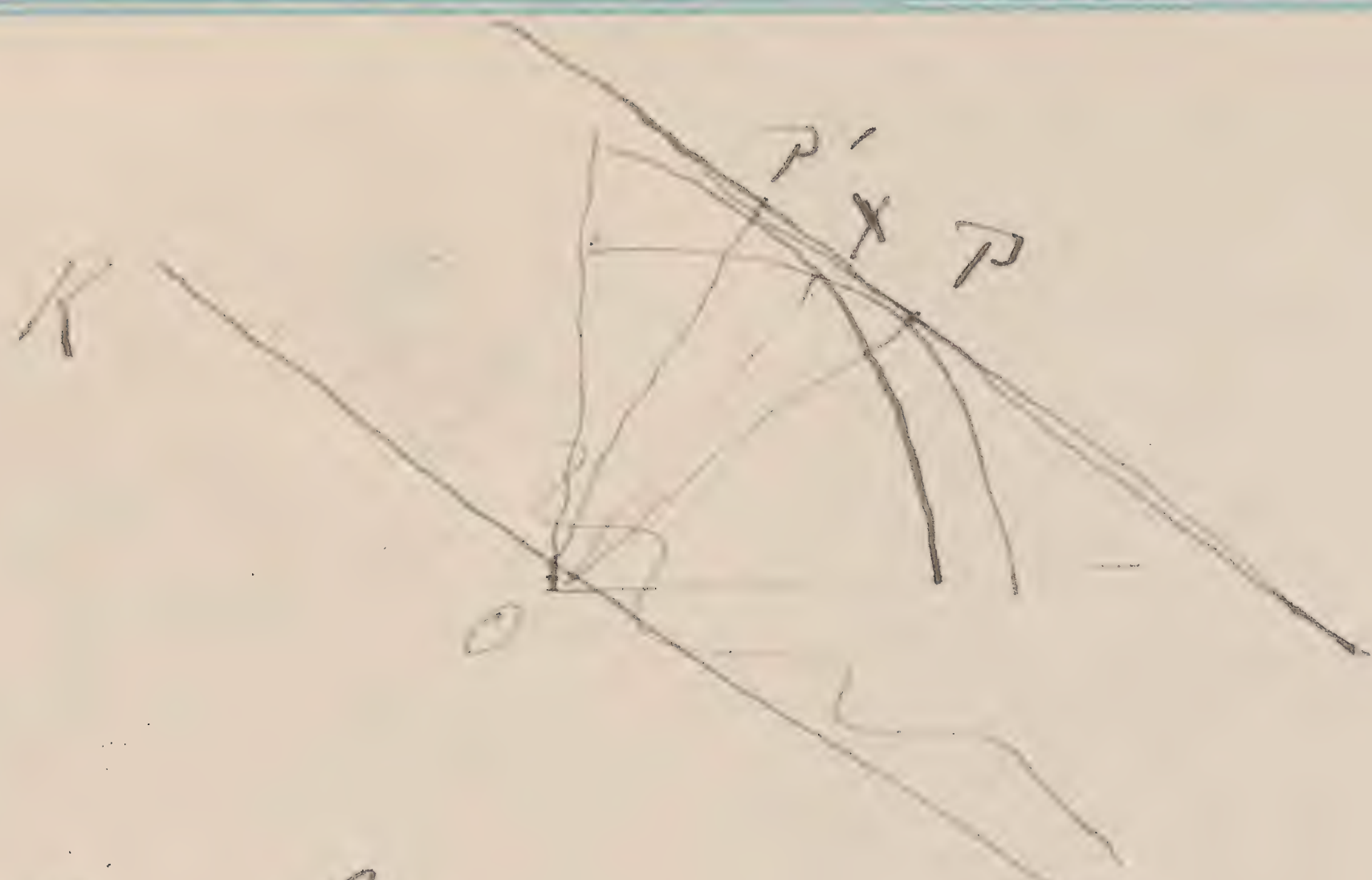
$ab =$ wave velocity for

vibrations in plane + to
paper
at: wave velocity of vibrations
in plane of paper.

OP and OP' are the
directions of the rays,
or that in wh. the shadow
would be cast - We have
two ray directions, or on
the ray is no longer +
to the wave front -

The wave and ray velocity
is the same in one case,
but different in the second

There is one case
in wh. the two wave
velocities are the same



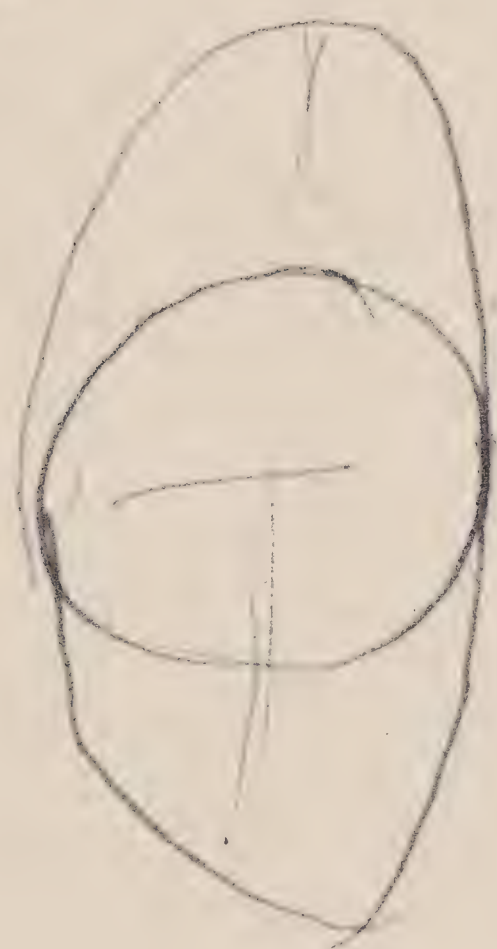
K is the surface of the
 crystal cut $\parallel$ to the
 line PP' the rays
 will be distinct. In
 OP is the optic axis.
 OX the secondary axis.

It may a special case
when the two optic axes
coincide - a uniaxial
crystal - The wave front
becomes

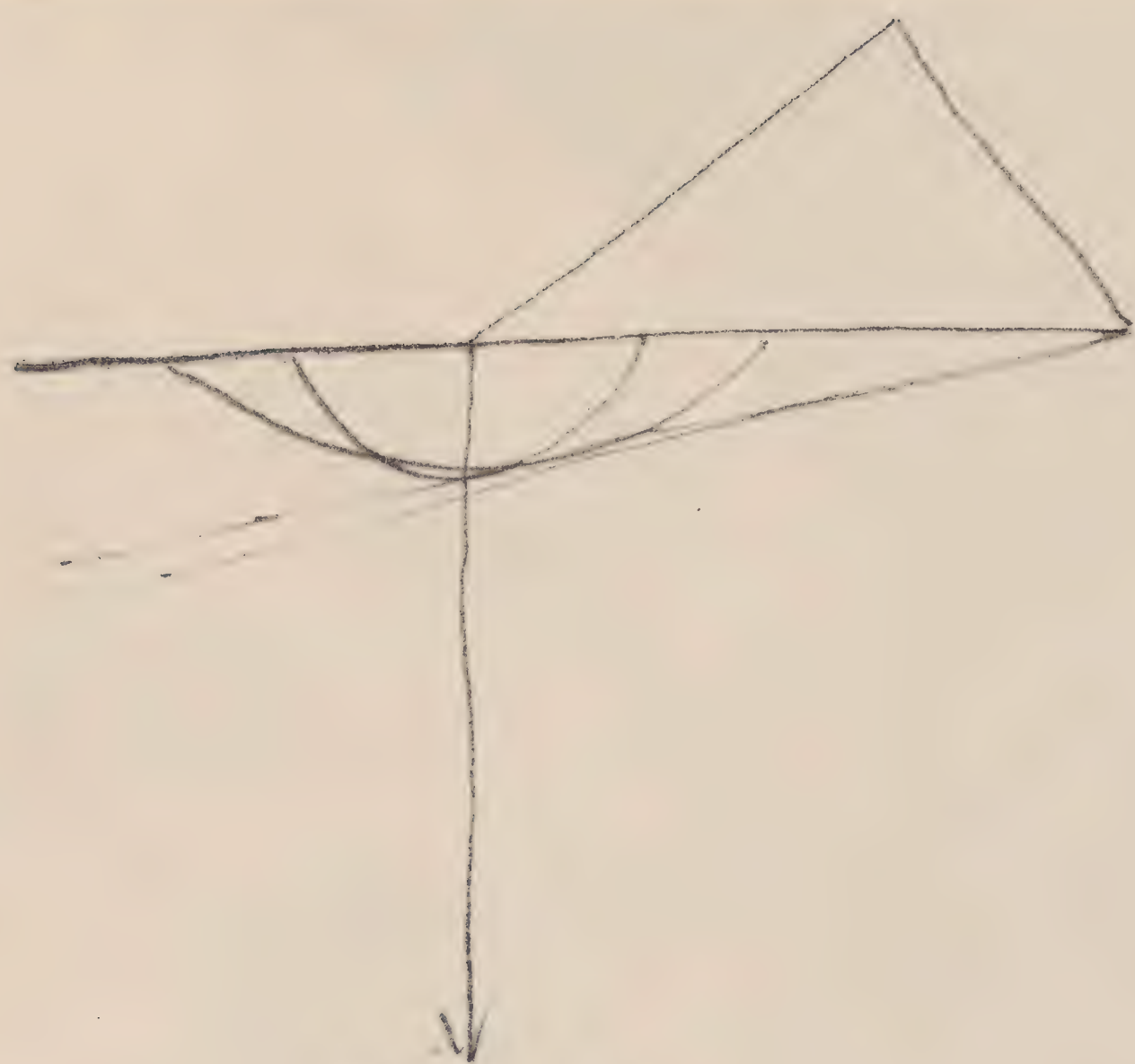


This is the case of positive
crystal - quartz etc

or



A negative crystal
as Iceland spar



Optic axis



Optic axis
 $\perp$ to plane

General case when optic
 axis is any direction



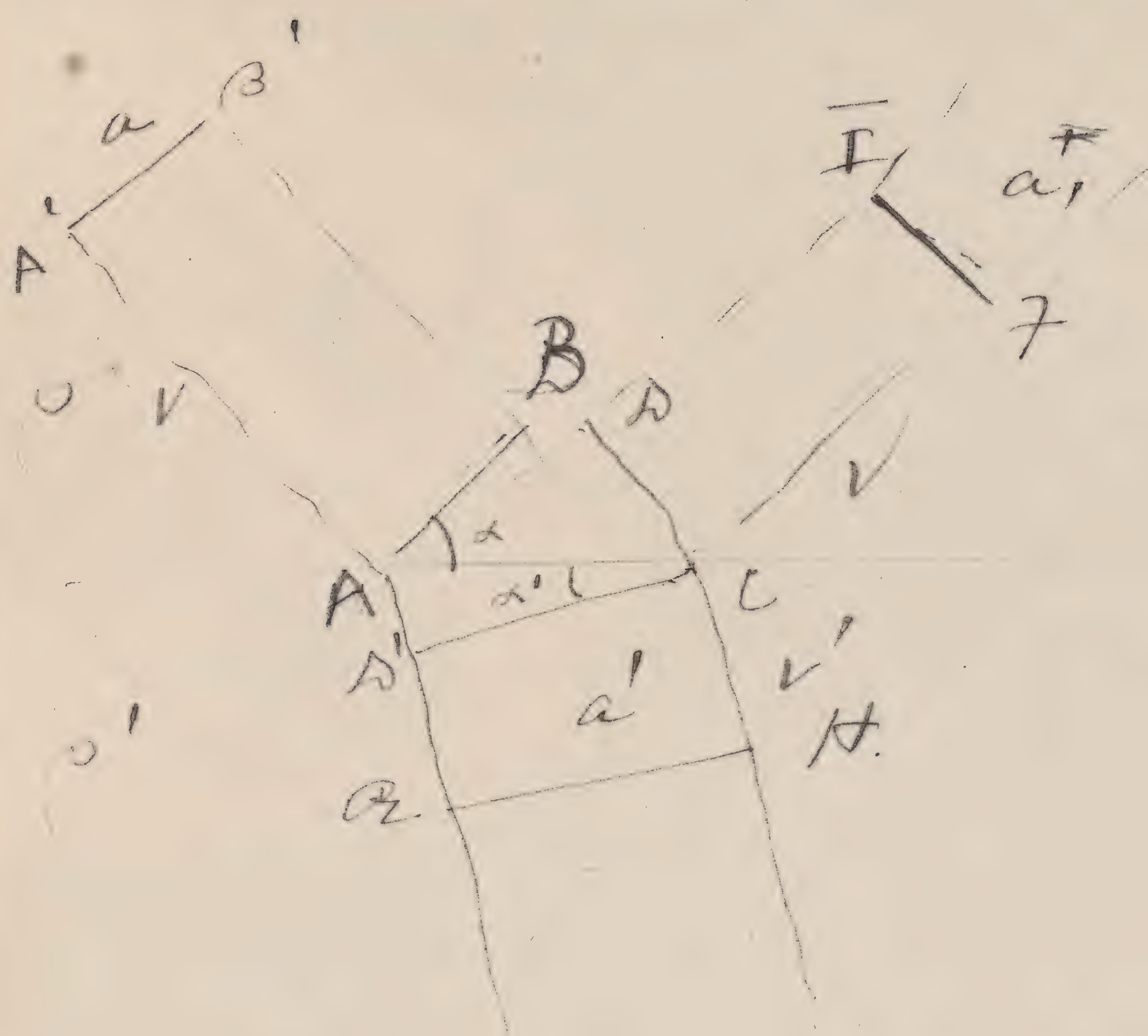
Nicol's prism, see pag. 336

Reflection

Snell's law -

reflected beam is plane polarized when the $\angle$ between the reflected and refracted beam is a rt. $\angle$.

This polar. is rather imperfect when reflected from only one surface -
Use a pile, a number of surfaces



Principle of conservation
of energy. shows.

formula for energy of
a vibrating mass

$$\frac{2\pi^2 a^2 M}{T^2}$$

a, a', a_1 = amplitudes of vibration

$\rho, \rho' =$ optical densities

Energy per unit vol. = $\frac{2\pi a^2}{T^2}$

Eq. of curve.

$$a^2 v \cos \alpha = a_1^2 p v \cos \alpha + a_1'^2 p' v' \cos \alpha'$$

I assume the elasticity
the same in the two media.

$$v = \sqrt{\frac{e}{\rho}}, \quad v' = \sqrt{\frac{e}{\rho'}}$$

$$\therefore v^2 \rho = v'^2 \rho'$$

$$\frac{\sin \phi}{\sin \phi'} = \frac{v}{v'}, \quad v' \sin \phi = v \sin \phi'$$

mult. eq. of course. $\phi = \alpha, \phi' = \alpha'$

$$a^2 v^2 \cos \phi \sin \phi' = a_1^2 v^2 \cos \phi \sin \phi' + a_2^2 v'^2 \cos \phi' \sin \phi$$

$$(a^2 - a_1^2) \cos \phi \sin \phi' = a_2^2 \cos \phi' \sin \phi$$

To determine relations we
must assume the
continuity of the medium
is maintained.

Shall take two cases.

1. Vibrations $\perp$ to plane
of drawing

$$a + a_1 = a'$$

$$a' = 2a \frac{\sin \phi' \cos \phi}{\sin(\phi + \phi')}$$

$$a_1 = -a \frac{\sin(\varphi - \varphi')}{\sin(\varphi + \varphi')}$$

In the expression $a + a_1 = a'$ is included the assumption that there is no change in phase

3. use vibration in plane of drawing - in direction AC

The component in horizontal direction must be same -

$$a \cos \varphi - a_1 \cos \varphi = a' \cos \varphi'$$

$$(a - a_1) \cos \varphi = a' \cos \varphi'$$

whence

$$a' = \frac{2a \cos \varphi \sin \varphi'}{\sin(\varphi + \varphi') \cos(\varphi - \varphi')}$$

$$a_1 = a \frac{\tan(\varphi - \varphi')}{\tan(\varphi + \varphi')}$$

Make use then of 1's

From these two cases can pass
to case of vibrations taking place
in any direction -

Let θ be the plane makes
with the y axis according

$$A \sin \theta = x$$

$$A \cos \theta = a'$$

$$A' \cos \theta' = 2A \cos \theta \frac{\sin \phi' \cos \phi}{\sin (\phi + \phi')}$$

$$A' \cos \theta' = A \cos \theta \frac{\sin (\phi - \phi')}{\sin (\phi + \phi')}$$

When $\phi + \phi' = 90^\circ$. Brewster's law.
 $\tan (\phi + \phi') = \infty$. For the reflected

ray, the vibrations in the
plane of the drawing have
a magnitude - hence this
reflected ray is plane polarized.
Vibrations being $\perp$ to plane of
drawing.

We have so far neglected
the continuity of ether on
reflections in the direction of
the normal

We also have that

$$(\alpha + \alpha_1) \sin \phi = \alpha' \sin \phi'$$

taking the product of this and
former

$$(\alpha^2 - \alpha_1^2) \cos \phi \sin \phi = \alpha'^2 \cos \phi' \sin \phi'$$

Comparing this with former
eqn. we find don't agree.

Handwritten scribbles and marks in the top left corner, possibly a signature or initials.

